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AD-823 637

ON THE ANALYSIS OF A CROSS-CORRELATION RECEIVER FOR THE DETECTION OF NOISE-LIKE SIGNALS

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AFLC, GAFB, N.Y., 24 Nov 67-123

FOREWORD

This report discusses the in-house effort accomplished under Project 4519, Task 451902 (System 760C).

The authors are indebted to Mr. Alfred S. Kobos for his careful reading of and help in rewriting the manuscript and to Captain Robert G. McLaughlin for his generous aid on many knotty programming difficulties.

This technical report has been reviewed by the Foreign Disclosure Policy Office (EMLI). It is not releasable to the Clearinghouse for Federal Scientific and Technical Information because it contains information embargoed from release to Sino-Soviet Bloc countries by AFR 400-10, "Strategic Trade Control Program."

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ABSTRACT

A common type of digital communication system is binary frequency shift keying (FSK) whereby every T seconds the transmitter sends a pulse of one of two frequencies. The receiver makes a decision (every T seconds) as to which frequency was transmitted. A sub-optimum receiver for this case obtains estimates of the two noise waveforms by passing received signals through filters centered at the sending frequencies and then cross-correlates these estimates with the received waveform. Two slightly different versions of this cross-correlator were considered, and the probability of error for each case was calculated. The results seem to agree with previous experimental work by Cossette and Wolf.

TABLE OF CONTENTS

1. INTRODUCTION

A common type of digital communication system is binary frequency shift-keying (FSK) whereby every T seconds the transmitter sends a pulse of one of two frequencies. The receiver then makes a decision (every T seconds) as to which frequency was transmitted. The method whereby the receiver makes this decision for various types of communications channels is the subject of this report.

The simplest model for a communications channel assumes that the input to the receiver is an attenuated version of the transmitted signal corrupted by additive Gaussian white noise (with zero mean and power spectral density $S(\omega) = No/2$ for all ω). This model has been thoroughly analyzed in the literature[1] and the receiver structure which leads to the minimum probability of error is known. Specifically, let us assume that the received waveform, r(t), is given as

$$r(t) = \left\{ \begin{array}{ll} \sqrt{2E/T} & \sin(\omega_o t + \theta_o) \\ & \text{or} \\ \sqrt{2E/T} & \sin(\omega_1 t + \theta_1) \end{array} \right\} + n(t) \quad o \leq t \leq T \tag{1}$$

where

(a) $\omega_o T = k2\pi$ k, an integer

(b) $\omega_1 T = p2\pi$ p, an integer not equal to k

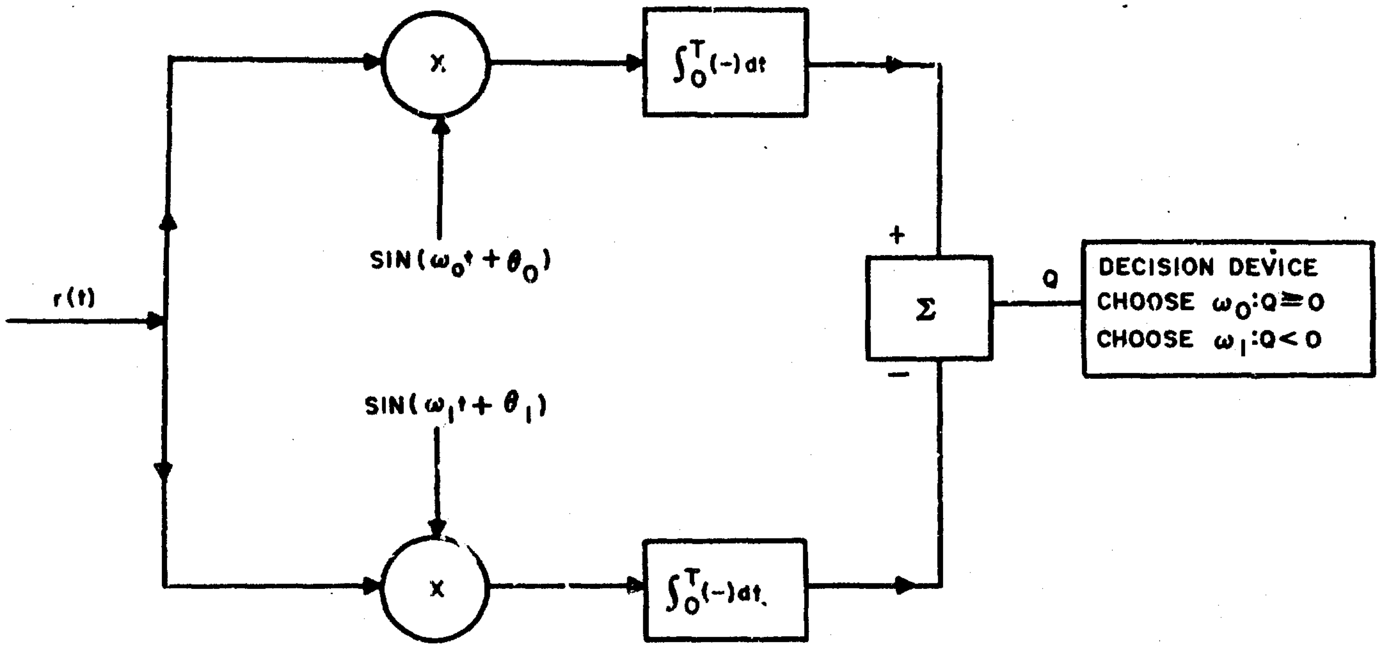
(c) $E\left\{n(t)\right\} = 0, \quad E\left\{n(t)n(t-\tau)\right\} = \frac{No}{2}\,\delta(\tau)$

and

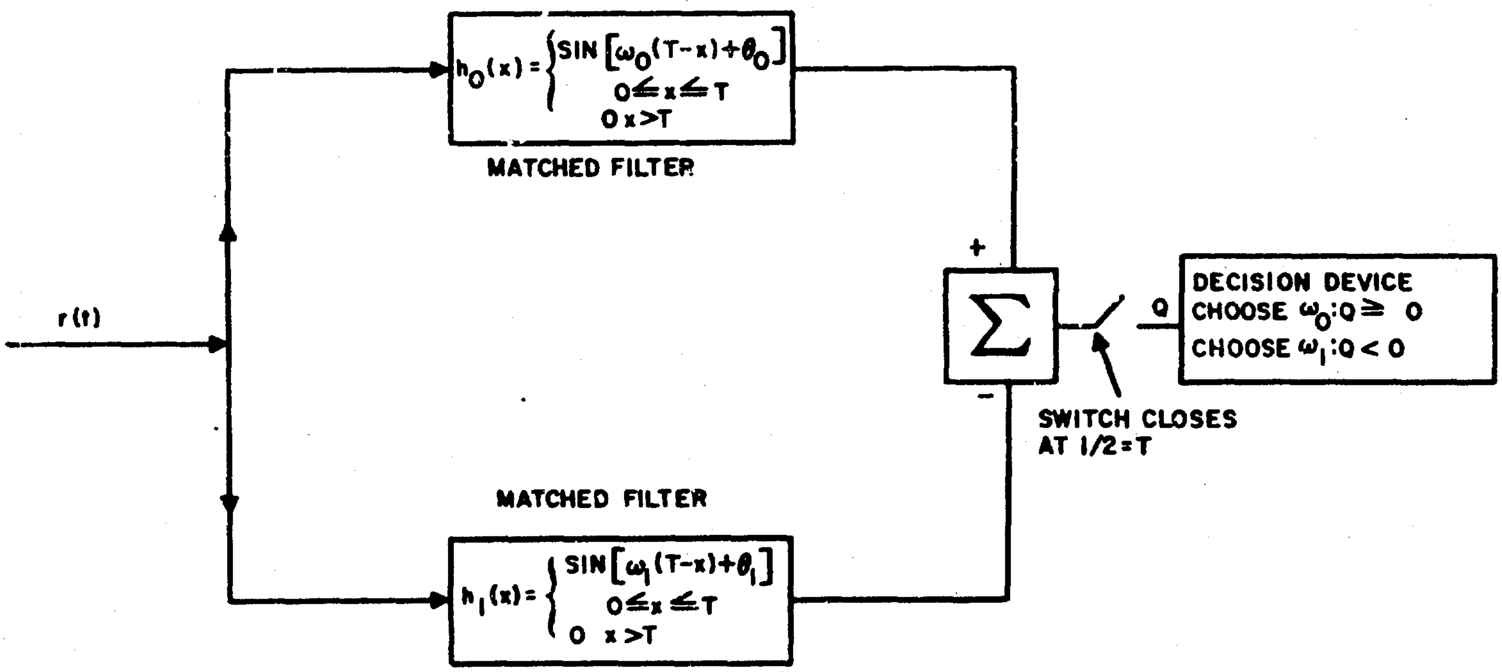
(d) The a priori probabilities of each sinusoid are equal.

Many receiver structures can be given all of which have identical performance. Two receiver structures which lead to the minimum probability of error for this simple channel model are given in Figure 1. The probability of error for these receivers is:

$$P_e = \frac{1}{\sqrt{2\pi}} \int_{\sqrt{\frac{E}{No}}}^{\infty} e^{-\frac{1}{2} Z^2} dz \tag{2}$$



(a) CORRELATION RECEIVER



(b) MATCHED FILTER RECEIVER

Figure 1. Two Realizations for Optimum Receiver for Additive Gaussian Noise Only

The above situation, termed coherent detection, may not be realistic for many reasons. One of these reasons is that it is assumed that the receiver has knowledge of the exact phase of both sinusoids. If the receiver knows nothing of the phase of the received sinusoids and it is either undesirable or impractical to assume that it has estimated this phase from previous pulses, then we can add the additional restriction that:

(e) The phases θ_1 and θ_2 are independent random variables, each having a probability density function which is uniform over the interval $(0, 2\pi)$.

Again the optimum receiver which leads to the minimum probability of error is known. Two such optimum receiver structures are given in Figure 2. The probability of error for these receivers is

$$P_e = \frac{1}{2} e^{-\frac{E}{2No}} \tag{3}$$

Continuing with the idea of making the mathematical model of the channel more general so that it applies to a wide class of channels, it is now assumed that there are statistical fluctuations in the amplitude of the FSK signals. The simplest situation to consider is the case where the amplitude $\sqrt{2E/T}$ is constant over any one pulse period but varies in a statistical manner from pulse to pulse. In that case, the previous receiver structures are still optimum but the average probability of error is given as

$$\overline{P_e} = \int_0^\infty (\text{probability of error without fading})\, p(E)dE \tag{4}$$

where p(E) is the probability density function of the signal energy E, which is a random variable. Amplitude fluctuations are usually accompanied by an unknown phase so (4) becomes:

$$\overline{P_e} = \int_0^\infty \frac{1}{2} e^{-\frac{E}{2No}} p(E)dE = M_E\left(-\frac{1}{2No}\right) \text{ (nonchoerent case)} \tag{4a}$$

where M_E (jv) is the characteristic function of the signal energy "E".

If, however, the phase is known (or a very good estimate is made of the phase) then (4) becomes

$$\overline{P_e} = \int_0^\infty \left(\frac{1}{\sqrt{2\pi}} \int_{\sqrt{E/No}}^\infty e^{-\frac{1}{2} Z^2} dz\right) p(E)dE \text{ (coherent case)} \tag{4b}$$

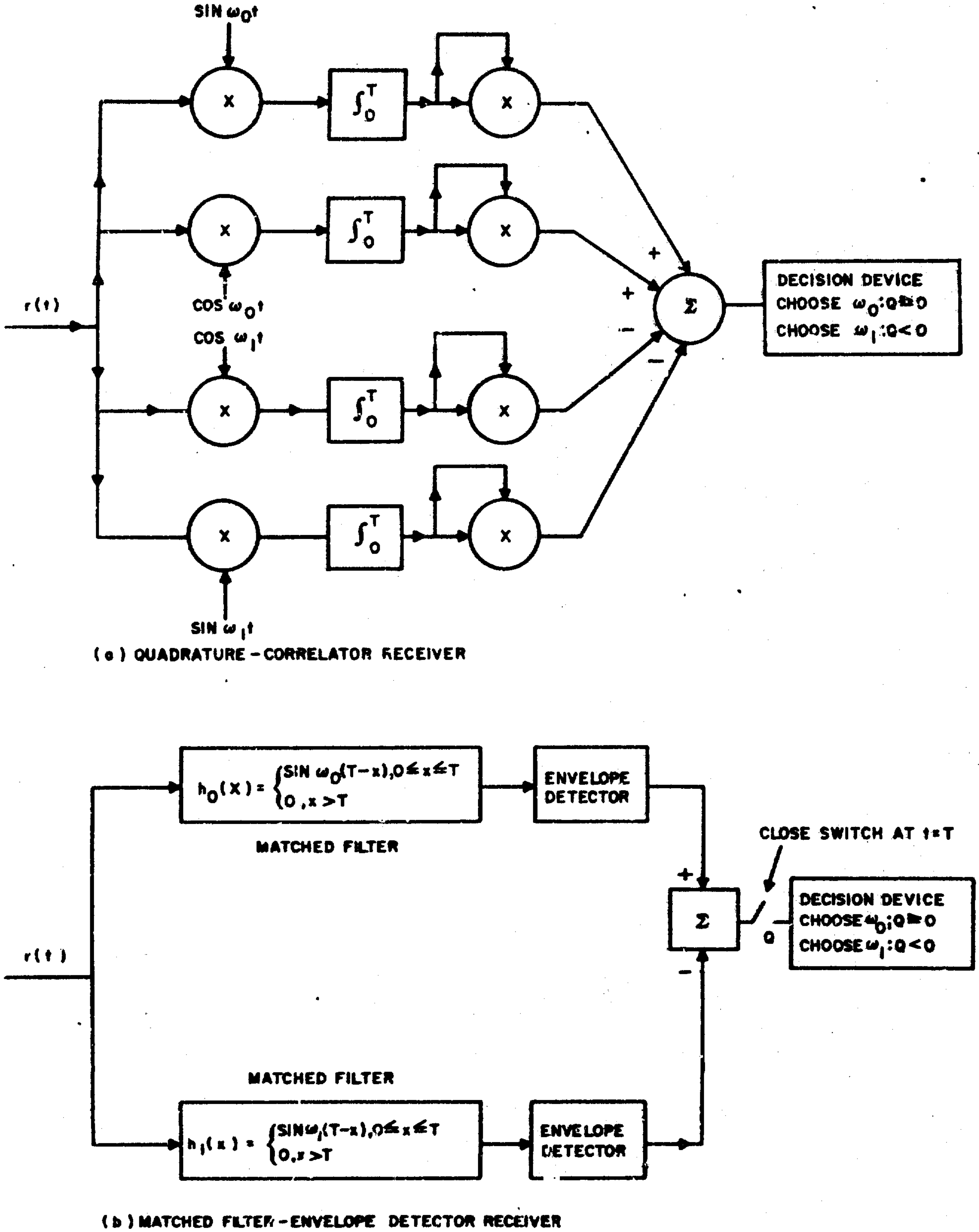


Figure 2. Two Realizations for Optimum Receiver for Additive Gaussian Noise and No Phase Information

A common type of fading experiment in practice is so-called Rayleigh fading where the envelope $\sqrt{2E/T}$ is distributed in accordance with the Rayleigh distribution. For that case, equations (4a) and (4b) become:

$$\overline{P}_e = \frac{1}{2 + \frac{\overline{E}}{No}} \quad \text{(noncoherent case)} \tag{5a}$$

and

$$\overline{P}_e = \frac{1}{2}\left[1 - \frac{\sqrt{\overline{E}/No}}{2 + \overline{E}/No}\right] \quad \text{(coherent case)} \tag{5b}$$

respectively, where $\overline{E}$ is the average value of the random variable E.

It is important to realize that equations (4) and (5) above do not apply to situations where the envelope fluctuates during one pulse period. Specifically, the above results do not apply to communications channels where the fading rate is of the same order or faster than the keying rate.

Mathematical expressions have been derived[3], the solutions of which give the optimum receiver for a fast fading case. Unfortunately, these equations have not been solved in general nor has the minimum probability of error been estimated. (Price[4] has derived the minimum probability of error for on-off keying with fast fading but not for FSK.) The aim of this report is to evaluate the performance of a particular (sub-optimum) receiver for this situation. The reason for the choice of the receiver chosen will be presented later.

2. MATHEMATICAL MODEL

In the model to be considered it is assumed that the receiver has as its input waveform a narrow band Gaussian signal centered either at frequency ω_o or ω_1 which is also corrupted by additive Gaussian white noise (again with zero mean and power spectral density $S_n(\omega) = No/2$ for all ω). In particular, the received waveform r(t) is given as

$$r(t) = \left\{\begin{matrix} n_o(t) \\ \text{or} \\ n_1(t) \end{matrix}\right\} + n(t) \qquad 0 \leq t \leq T \tag{6}$$

where

(a) $n_o(t)$, $n_1(t)$ and $n(t)$ are independent Gaussian processes all with zero mean

(b) $E\left\{n(t)n(t-\tau)\right\} = \frac{No}{2}\,\delta(\tau)$

(c) $E\left\{n_o(t)n_o(t-\tau)\right\} = R_o(\tau)$

$E\left\{n_1(t)n_1(t-\tau)\right\} = R_1(\tau)$

(d) The a priori probabilities of $n_1(t)$ and $n_o(t)$ are equal.

The receiver must operate on the received waveform r(t) during the interval (0, T) and make a decision whether $n_o(t)$ or $n_1(t)$ was present during that interval.

The physical reasoning for such a mathematical model is that when a sinusoid is transmitted over a scatter communications channel, it travels over many different paths, each path introducing amplitude, phase and perhaps frequency changes. The sum of the signals from all these paths then (from Central Limit Theorem arguments) can be considered as a narrow band Gaussian process with center frequency given by the frequency of the transmitted carrier. An artificial channel which was constructed and exhibits this type of perturbation was the Needles belt(6).

An intuitive argument might suggest that a natural method for deciding between the two noise sources is to estimate the energy in the received waveform in the two narrow frequency bands centered at ω_o and ω_1 and then choose that frequency having the largest energy. A breadboard simulation of such a system was reported by Cossette and Wolf(7). Theoretical analyses of such a system have been made by Jacobs(8) (for one type of spectrum) and by Kobos and Meyer(9).

Another sub-optimum receiver is suggested by the following ideas. Note that the correlator receiver given in Figure 1 cross-correlated the received waveform with stored replicas of the transmitted signals. Thus it would seem that a logical design for a receiver would be to obtain estimates of the two noise waveforms $n_o(t)$ and $n_1(t)$ by passing the received signal through filters centered at ω_o and ω_1 and then cross-correlating these estimates with the received waveform r(t). Such a receiver, called a cross-correlator receiver, is shown in Figure 3. This receiver is analyzed in the remaining sections of this report.

It should be clearly understood that no claim for optimality is made for this receiver. However, it is interesting to note that one form for the block diagram of the optimum receiver (which leads to minimum probability of error) is similar to this receiver(10). The optimum receiver, however, utilizes time varying filters in place of narrow band filters. The time-varying impulse responses for these filters are not known, but are only known to be the solutions to certain integral equations. Furthermore, since time varying filters may be difficult to build, there appears to be adequate justification for considering this simpler, but sub-optimum, receiver.

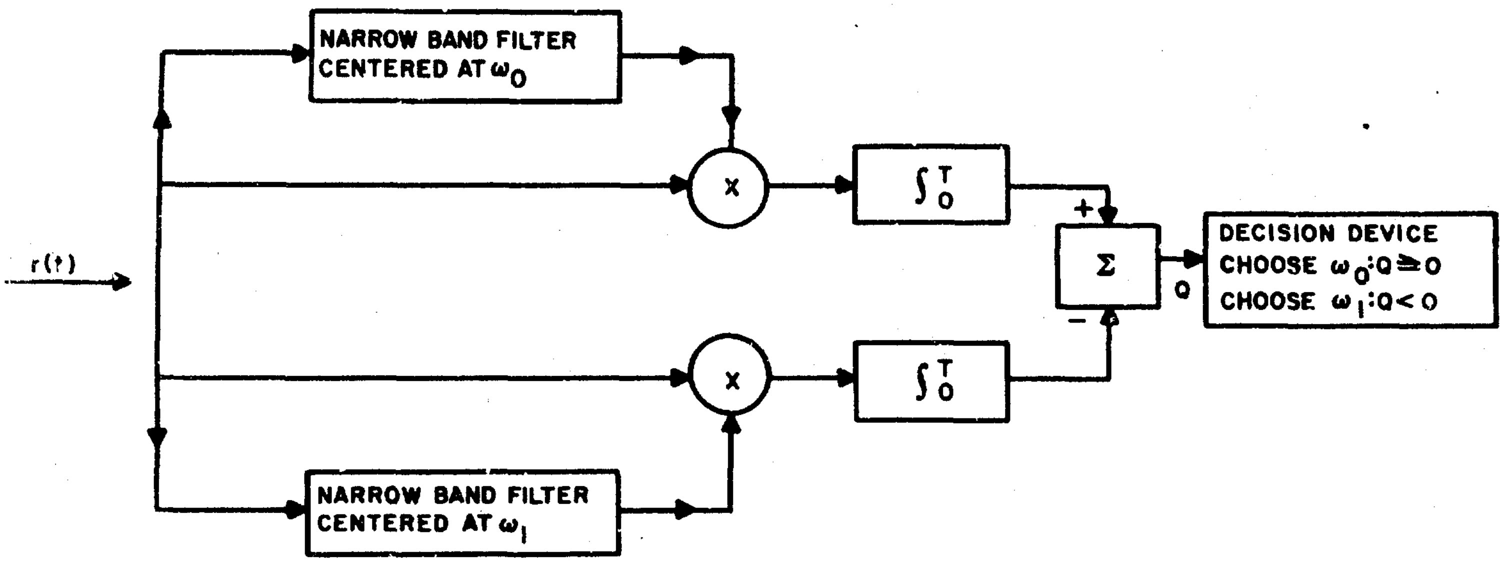


Figure 3. Cross-Correlator Sub-Optimum Receiver for Noise-Like FSK Reception

3. MATHEMATICAL ANALYSIS

In the following analysis specific forms are assumed for the transfer functions of the filters in Figure 3, as well as for the autocorrelation functions of the noise processes. Although the problem could be analyzed in greater generality, this approach was taken since a more general analysis would be more complicated and the salient points are illustrated in the analysis which follows.

Specifically, we assume that the noise processes $n_0(t)$ and $n_1(t)$ have autocorrelation functions identical to those which would result by passing white noise through single-tuned, high Q, RLC filters centered at ω_0 and ω_1 respectively. Thus, if we write $n_0(t)$, and $n_1(t)$ as

$$n_i(t) = x_i(t) \cos \omega_i t + y_i(t) \sin \omega_i(t) \qquad i = 0, 1 \tag{7}$$

Then

$$R_{x_i x_i} = R_{y_i y_i} (\tau) = e^{-\alpha |\tau|} \qquad i = 0, 1 \tag{8}$$

$$R_{x_i y_i} (\tau) = o \qquad i = 0, 1 \tag{9}$$

Note that "S" is the power in $x_1(t)$ and $y_1(t)$. As a consequence of equation (9), however, it is also the power in $n_1(t)$. Furthermore, the impulse response of the receiver filters are:

<u>Narrow band filter centered at ω_o</u>

$$h_o(x) = \begin{cases} e^{-\alpha x} \cos \omega_o x & x \geq 0 \\ 0 & x < 0 \end{cases} \tag{10}$$

<u>Narrow band filter centered at ω_1</u>

$$h_1(x) = \begin{cases} e^{-\alpha x} \cos \omega_1 x & x \geq 0 \\ 0 & x < 0 \end{cases} \tag{11}$$

That is, these receiver filters are just RLC, high Q filters centered at ω_o and ω_1, respectively. Furthermore, we assume that the center frequencies ω_o and ω_1 are separated far enough, so that there is no output of filter centered at ω_1 due to the narrow band noise $n_o(t)$ (centered at ω_o) and there is no output of filter centered at ω_o due to the narrow band noise $n_1(t)$ (centered at ω_1). Of course, both filters have outputs due to the additive white noise $n(t)$.

Two slightly different versions of the problem will be considered. They are described below and referred to as case 1 and case 2. In both situations we will assume that the received waveform actually contained the narrow band noise $n_o(t)$ plus the additive Gaussian white noise $n(t)$. We then calculate the probability of error as the probability that the receiver decides that $n_1(t)$ was present. This, of course, is just one type of error that the receiver could make but due to the symmetry of the problem, this error probability is equal to the overall error probability.

4. CASE 1

Consider the receiver shown in Figure 4, where the received waveform is given as

$$r(t) = n_o(t) + n(t). \qquad 0 \leq t \leq T \tag{12}$$

The probability of error for this receiver is then

$$P_e = P_r [Q < 0] = P_r [Q_1 > Q_o]. \tag{13}$$

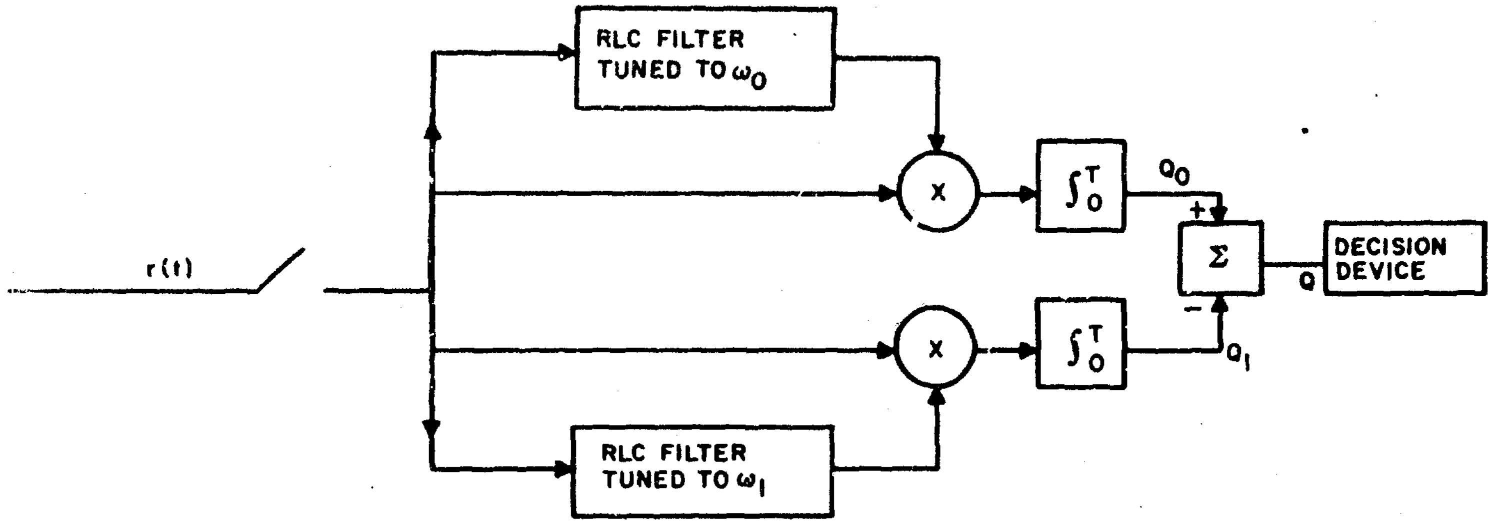


Figure 4. Correlator Receiver for Case 1.

As in equation (7), we write

$$n_o(t) = x_o(t) \cos \omega_o t + y_o(t) \sin \omega_o t \tag{14}$$

The switch in the front end of the receiver, which closes at $t = 0$, is included to specifically indicate that the receiver filters are inert (have no initial conditions) at the start of the keying interval. (In practice the energy in these filters would have to be dumped at the end of every pulse period.)

To calculate the output voltage Q_0, we write the white noise $n(t)$ as*

$$n(t) = x_n(t) \cos \omega_o t + y_n(t) \sin \omega_o t \tag{15}$$

where $$E[x_n(t)x_n(t - \tau)] = E[y_n(t)\, y_n(t - \tau)] = \frac{No}{2}\, \delta(\tau) \tag{16}$$

and $$E[x_n(t)\, y_n(t - \tau)] = o \tag{17}$$

*The complications of writing white noise in the form given in equation (15) are discussed by Helstrom[11].

Since $n_o(t)$ and $n(t)$ are statistically independent processes, we then have

$$r(t) = x_r(t) \cos \omega_o t + y_r(t) \sin \omega_o t \tag{18}$$

where

$$E[x_r(t) x_r(t - \tau)] = Se^{-\alpha|\tau|} + \frac{No}{2} \delta(\tau) = E[y_r(t) y_r(t - \tau)], \tag{19}$$

and

$$E[x_r(t) y_r(t - \tau)] = o \tag{20}$$

Now

$$Q_o = \int_o^T \left\{ \int_o^t e^{-\alpha(t-\eta)} \cos \omega_o(t - \eta) [x_r(\eta) \cos \omega_o \eta + y_r(\eta) \sin \omega_o \eta] \, d\eta \right\} [x_r(t) \cos \omega_o t + y_r(t) \sin \omega_o t] \, dt \tag{21}$$

But if $f(x, y) = f(y, x)$, then

$$\int_o^T [\int_o^y f(x, y) \, dx] \, dy = \frac{1}{2} \int_o^T \int_o^T f(x, y) \, dx \, dy \tag{22}$$

so that Q_o can be written as

$$Q_o = \frac{1}{2} \int_o^T \int_o^T e^{-\alpha(t-\eta)} \cos \omega_o(t - \eta) [x_r(\eta) \cos \omega_o \eta + y_r(\eta) \sin \omega_o \eta] [x_r(t) \cos \omega_o t + y_r(t) \sin \omega_o t] \, dt \, d\eta \tag{23}$$

After expanding all trigonometric products in sums of trigonometric functions, we can ignore all terms which involve $\cos \omega_o t$, $\sin \omega_o t$, $\cos \omega_o \eta$, or $\sin \omega_o \eta$, since their contributions to Q_o will be negligible after performing the integration. We are then left with the expression

$$Q_o = \frac{1}{8} \int_o^T \int_o^T e^{-\alpha|t - \eta|} [x_r(t) x_r(\eta) + y_r(t) y_r(\eta)] \, dt \, d\eta \tag{24}$$

Let us now consider the orthonormal set of functions $\phi_i(t), i = 1, 2, \ldots$ which are solutions to the integral equation

$$\int_o^T e^{-\alpha|t-\tau|} \phi_i(\tau)\, d\tau = \lambda_i \phi_i(t) \quad o \leq t \leq T \tag{25}$$

where

$$\int_o^T \phi_i(t)\, \phi_j(t)\, dt = \delta_{ij} \tag{26}$$

If we expand $x_r(t)$ and $y_r(t)$ in terms of these functions $\phi_i(t)$ to obtain

$$x_r(t) = \sum_{j=1}^{\infty} x_j \phi_j(t) \tag{27}$$

$$y_r(t) = \sum_{j=1}^{\infty} y_j \phi_j(t) \tag{28}$$

then it is easy to show that

$$E[x_i x_j] = (S\lambda_j + \frac{No}{2})\, \delta_{ij} = E[y_i y_j] \tag{29}$$

We can thus rewrite Q_o in terms of the coefficients x_j, y_j and the functions $\phi_j(t)$ as

$$Q_o = \frac{1}{8} \int_o^T \int_o^T e^{-\alpha|t-\eta|} \left[\sum_{j=1}^{\infty} (x_j + y_j)\, \phi_j(\eta)\right]\left[\sum_{k=1}^{\infty} (x_k + y_k)\, \phi_k(t)\right] dt\, d\eta \tag{30}$$

But making use of Equations (25) and (26), this becomes

$$Q_o = \frac{1}{8} \sum_{j=1}^{\infty} \lambda_j (x^2_j + y^2_j) \equiv \sum_{j=1}^{\infty} \epsilon_j \tag{31}$$

where $$\epsilon_j = \frac{1}{8}\, (x^2_j + y^2_j)\, \lambda_j \tag{32}$$

Since all processes under consideration are Gaussian, x_j and y_j are Gaussian random variables so that ϵ_j is chi-square distributed with mean value

$$E[\epsilon_j] = \frac{1}{4}\lambda_j(S\lambda_j + \frac{No}{2}) \equiv \bar{\epsilon}_j \tag{33}$$

Thus the probability density function of ϵ_j is given as

$$p(\epsilon_j) = \frac{1}{\bar{\epsilon}_j} e^{-\epsilon_j/\bar{\epsilon}_j}, \quad \epsilon_j \geq o, \tag{34}$$

and its characteristic function $M_{\epsilon_j}(jv)$ is

$$M_{\epsilon_j}(jv) = \int_o^{\infty} p(\epsilon_j) e^{+jv\epsilon_j} d\epsilon_j = \left(\frac{1}{1 - jv\bar{\epsilon}_j}\right) \tag{35}$$

Since the ϵ_j are all statistically independent, the characteristic function of Q_o is then

$$M_{Q_o}(jv) = \prod_j \frac{1}{(1 - jr\bar{\epsilon}_j)} \tag{36}$$

where $\bar{\epsilon}_j$ is given in Equation (33).

Let us now concentrate on calculating the statistics of Q_1. Since the center frequency of this filter is ω_1, it is convenient to write the noise as

$$n(t) = x'_n(t) \cos \omega_1 t + y'_n(t) \sin \omega_1 t \tag{37}$$

so that the input to the filter becomes

$$r(t) = x_o(t) \cos \omega_o t + y_o(t) \sin \omega_o t + x'_n(t) \cos \omega_1 t + y'_n(t) \sin \omega_1 t \tag{38}$$

The output Q_1 is then

$$Q_1 = \int_o^T \left\{ \int_o^t e^{-\alpha(t-\eta)} \cos \omega_1 (t-\eta) [r(\eta)] d\eta \right\} r(t) dt \tag{39}$$

where r(t) is as given in Equation 38. Substituting Equation (38) into (39) (twice) and ignoring all terms which have sinusoidal variations, results in

$$Q_1 = \frac{1}{4} \int_o^T \int_o^t e^{-\alpha(t-\eta)} \left[x'_n(\eta)\, x'_n(t) + y'_n(\eta)\, y'_n(t) \right] d\eta \; dt \tag{40}$$

or

$$Q_1 = \frac{1}{8} \int_o^T \int_o^T e^{-\alpha|t-\eta|} \left[x'_n(\eta)\, x'_n(t) + y'_n(\eta)\, y'_n(t) \right] d\eta \; dt \tag{41}$$

Expanding $x'_n(t)$ and $y'_n(t)$ in terms of the functions $\phi_i(t)$ previously defined, as

$$x'_n(t) = \sum_{j=1}^{\infty} x'_j \; \phi_j(t) \tag{42}$$

$$y'_n(t) = \sum_{j=1}^{\infty} y'_j \; \phi_j(t) \tag{43}$$

and substituting into Equation (41) yields

$$Q_1 = \frac{1}{8} \sum_{j=1}^{\infty} \lambda_j \left[(x'_j)^2 + (y'_j)^2 \right] \equiv \sum_{j=1}^{\infty} \epsilon'_j \tag{44}$$

By a similar set of steps to that which led to Equation (36) we obtain

$$M_{Q_1}(jv) = \prod_j \frac{1}{(1 - jv\overline{\epsilon'_j})} \tag{45}$$

where

$$\overline{\epsilon'_j} = \frac{1}{8} N_o \lambda_j \tag{46}$$

We are now in a position to calculate the probability of error. Returning to Equation (13) we see that the probability of error is given as

$$Pe = P_r \left[Q_1 > Q_o \right] = \int_o^{\infty} p(Q_o) \left[\int_{Q_o}^{\infty} p(Q_1)\, d\, Q_1 \right] dQ_o \tag{47}$$

But the probability density functions $p(Q_i)$ and the characteristic functions M_{Q_i} (jv) are related by the equation

$$P(Q_i) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} M_{Q_i} (jv)\, e^{-jvQ_i}\, dv = \frac{1}{2\pi} \int_{-\infty}^{+\infty} M^*_{Q_i} (jv)\, e^{+jvQ_i}\, dv \tag{48}$$

Substituting one form of Equation (48) into Equation (47) we obtain

$$Pe = \int_o^{\infty} p(Q_o) \left[\int_{Q_o}^{\infty} \left[\frac{1}{2\pi} \int_{-\infty}^{+\infty} M^*_{Q_1} (jv)\, e^{+jvQ_1}\, dv \right] dQ_1 \right] dQ_o \tag{49}$$

or

$$Pe = \int_o^{\infty} p(Q_o) \left[\frac{1}{2\pi} \int_{-\infty}^{+\infty} M^*_{Q_1} \frac{e^{+j\, vQ_o}}{-jv}\, dv \right] dQ_o, \text{ if Re } (j\, v) < o. \tag{50}$$

Performing the integration with respect to Q_o yields

$$Pe = - \frac{1}{2\pi j} \int_{-\infty}^{+\infty} \cdot \frac{M_{Q_o} (jv)\, M^*_{Q_1} (jv)\, dv}{V} \quad \text{if Re } (j\, v) < o \tag{51}$$

Finally, we can rewrite Equation (51) in terms of a complex variable s, as

$$Pe = -\frac{1}{2\pi j} \int_C \frac{M_{Q_o} (S)\, M^*_{Q_1} (S)}{S}\, ds \tag{52}$$

where C is a contour along the imaginary axis but to the left of all the poles on this axis.

From Equations (36) and (45) we have that

$$M_{Q_o}(S) = \lim_{N \to \infty} \prod_{j=1}^{N} \frac{1}{(1 - S\,\bar{\epsilon}_j)} \tag{53}$$

and

$$M^*_{Q_1}(S) = \lim_{N \to \infty} \prod_{j=1}^{N} \frac{1}{(1 - S\,\bar{\epsilon}'_j)} \tag{54}$$

where

$$\bar{\epsilon}_j = \frac{1}{4} \lambda_j \left(S\lambda_j + \frac{No}{2}\right) \tag{55}$$

and

$$\overline{\epsilon'_j} = \frac{1}{8} \lambda_j N_o \tag{56}$$

Thus we can write (assuming that we can take the limit after integrating)

$$Pe = \lim_{N \to \infty} -\frac{1}{2\pi j} \int_C \frac{ds}{S \prod_{i=1}^{N} (1 - S\,\bar{\epsilon}_j)(1 + S\,\bar{\epsilon}_j)} \tag{57}$$

Defining the pole locations S_j and S'_j as

$$S_j = \frac{1}{\bar{\epsilon}_j} = \frac{4}{\lambda_j \left(S\lambda_j + \frac{No}{2}\right)} \tag{58}$$

$$S'_j = \frac{1}{\overline{\epsilon'_j}} = \frac{8}{\lambda_j N_o} \tag{59}$$

we then have

$$Pe = \lim_{N \to \infty} \frac{(-1)^{N+1}}{2\pi j} \int_C \frac{\prod_{i=1}^{N} S_i S'_i \, ds}{S \prod_{i=1}^{N} (S - S_i)(S + S'_i)} \tag{60}$$

In order to perform the integration indicated in Equation (60) we note the pole plot shown in Figure 5. (The eigenvalues, λ_i, have been ordered such that $\lambda_1 > \lambda_2 > \lambda_3 . . .$). Let us write Equation (60) as

$$Pe = \lim_{N \to \infty} \frac{1}{2\pi j} \int_C f_N (S)\, ds \tag{61}$$

Then from the residue theorem, the probability of error is the sum of the residues of $f_N(s)$ for the left half plane poles. The residue of $f_N(s)$ at $S = -S'_j$ is given as:

$$S = -S'_j = (-1)^{N+1} \frac{\prod_{i=1}^{N} S_i S'_i}{(-S_j) \prod_{i=1}^{N} (-1)^N (S'_j + S_i) \prod_{\substack{i=1 \\ i \neq j}}^{N} (S'_i - S'_j)} \tag{62}$$

or

$$\text{residue of } f_N(s) \text{ at } \quad S = -S'_j = \frac{1}{\left(1 + \frac{S'_j}{S_j}\right) \prod_{\substack{i=1 \\ i \neq j}}^{N} \left(1 + \frac{S'_j}{S_i}\right)\left(1 - \frac{S'_j}{S_i}\right)} \tag{63}$$

the probability of error is then

$$Pe = \lim_{N \to \infty} \sum_{j=1}^{N} \frac{1}{\left(1 + \frac{S'_j}{S_j}\right) \prod_{\substack{i=1 \\ i \neq j}}^{N} \left(1 + \frac{S'_j}{S_i}\right)\left(1 - \frac{S'_j}{S'_i}\right)} \tag{64}$$

Finally, substituting the values for S_j and S'_j given in Equations (58) and (59) yields

$$Pe = \lim_{N \to \infty} \sum_{j=1}^{N} \frac{1}{2\left(1 + \frac{S\lambda_j}{N_o}\right) \prod_{\substack{i=1 \\ i \neq j}}^{N} \left[1 + \left(1 + \frac{2S\lambda_i}{N_o}\right) \frac{\lambda_i}{\lambda_j}\right]\left[1 - \frac{\lambda_i}{\lambda_j}\right]} \tag{65}$$

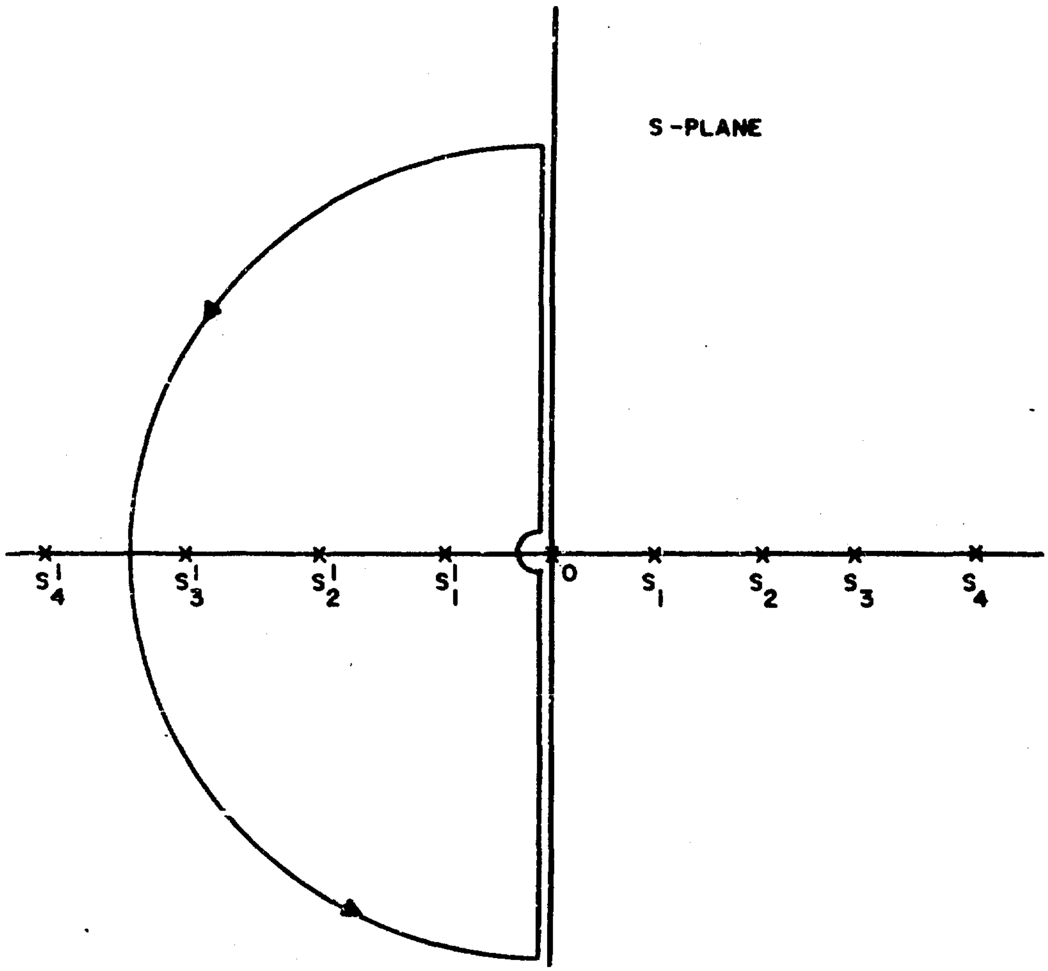


Figure 5. Pole Plot and Path of Contour Integration for Equation (60)

Equation (65) is the desired result. Note that this equation gives the probability of error in terms of eigenvalues λ_i of the integral equations given by Equation (25) and also the ratio S/No where S is the power in the "signal", $n_0(t)$ and No is the noise per cycles/second of bandwidth for the additive white noise. A computer was used to calculate Pe for various values of (αT) as ST/N_0 varied over a range of values. These results are discussed in a later section.

5. CASE 2

The second case differs from the first in the way in which the additive white noise is treated. Now it is assumed that the cross-correlations are each preceded by additional RLC filters which are not dumped at the end of each pulse period. Furthermore,

the spectrum of the noise $n_0(t)$ is assumed to be wide compared to the bandwidth of the RLC filter centered at ω_0 so that it can be considered as a white noise input. However, the separation between ω_0 and ω_1 is assumed wide enough so that $n_0(t)$ can be ignored as an input to the filters centered at ω_1. The actual receiver structure and the circuit which is claimed as equivalent for calculation purposes are shown in Figure (6a) and (6b) respectively.

The analysis for Case 2 is performed in a similar fashion to that of Case 1. The resultant expression for probability of error is

$$Pe = \lim_{N \to \infty} \sum_{j=1}^{N} \frac{1}{(1+\mu) \prod\limits_{\substack{i=1 \\ i \neq j}}^{N} \left(1 + \mu \frac{\lambda_i^2}{\lambda_j^2}\right)\left(1 - \frac{\lambda_i^2}{\lambda_j^2}\right)} \tag{66}$$

where

$$\mu = \frac{\text{signal power + noise power}}{\text{noise power}} \tag{67}$$

and again λ_j are the eigenvalues of the integral equation given in Equation (25). Computer results are also given for Case 2 in the next section.

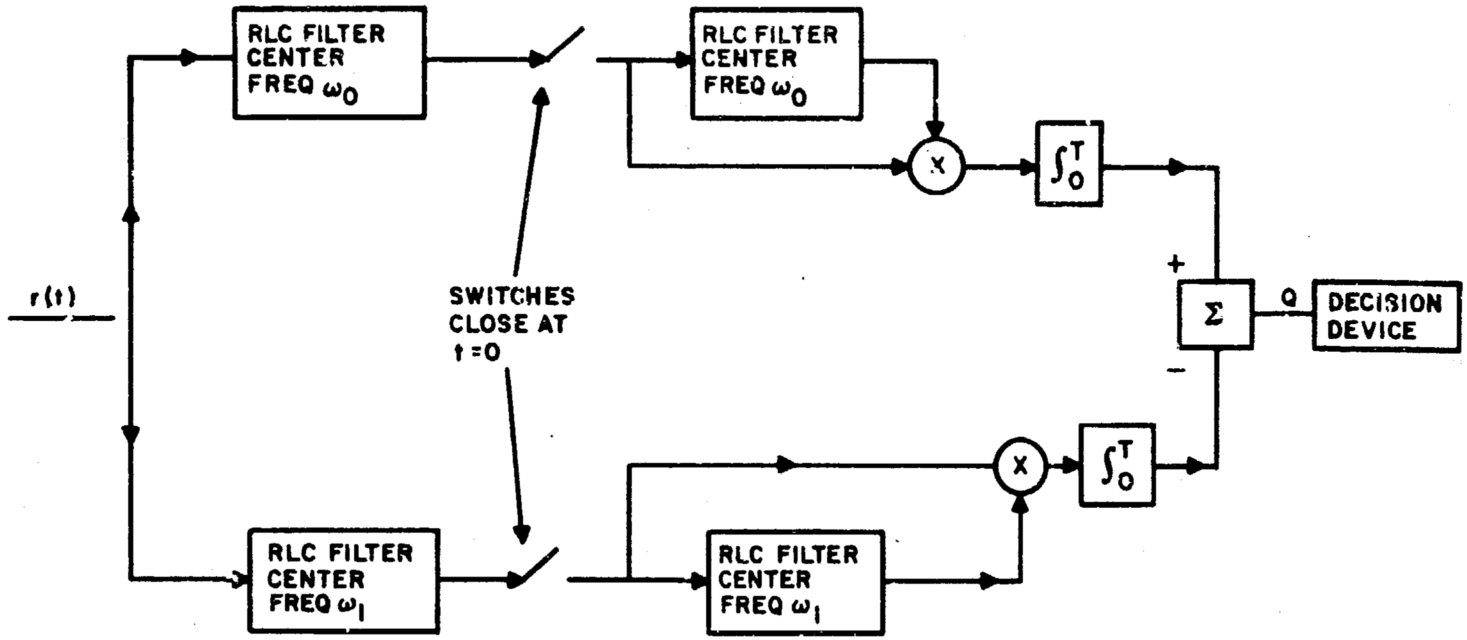
6. COMPUTER EVALUATION OF ERROR PROBABILITIES

In order to compare Cases 1 and 2, a common set of parameters must be defined. In the introduction, it was seen that error probability expressions were always expressed in terms of the ratio E/No where

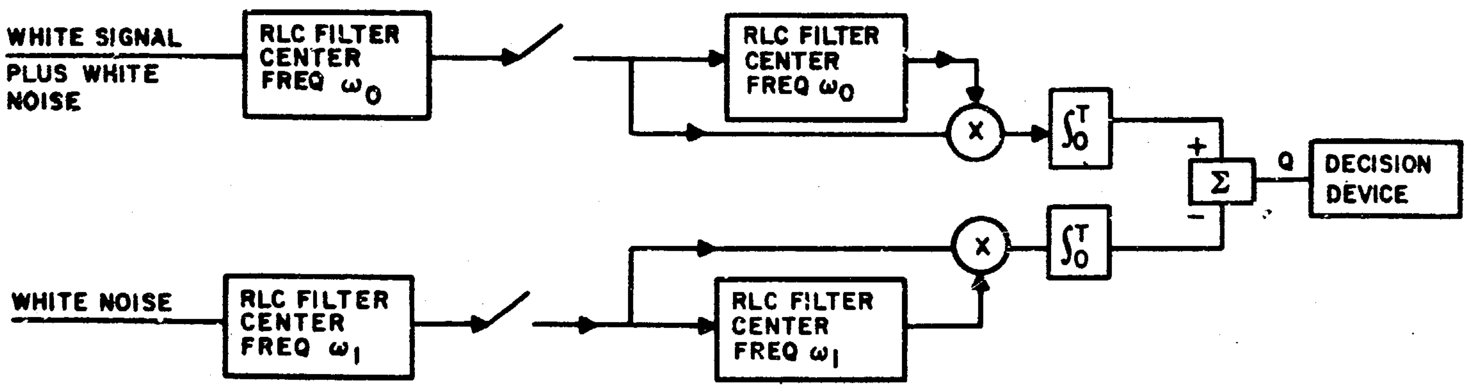
$$\frac{E}{No} = \frac{\text{energy in the signal for one pulse period}}{\text{noise power density}} \tag{68}$$

This ratio will also be used for the two cases considered here. The second parameter was chosen as the dimensionless "time-bandwidth" product, β, defined as

$$\beta = \frac{\alpha T}{2} \tag{69}$$



(a) ACTUAL CIRCUIT FOR CASE 2.



(b) EQUIVALENT CIRCUIT FOR CASE 2.

Figure 6. Receiver for Case 2

Furthermore, it is easily shown that the expressions for error probability depend only on the parameter β and not on the individual values of α and T. Thus we can arbitrarily set α equal to 10 so that

$$\beta = 5T$$

In terms of these two parameters (β and E/N_o) the two expressions for probability of error become

Case 1

$$P_e = \lim_{N\to\infty} \sum_{j=1}^{N} \frac{1}{2\left(1+\frac{5E}{N_o}\frac{\lambda_j}{\beta}\right)\prod_{\substack{i=1\\ i\neq j}}^{N}\left[1+\left(1+\frac{10E}{N_o}\frac{\lambda_i}{\beta}\right)\frac{\lambda_i}{\lambda_j}\right]\left[1-\frac{\lambda_i}{\lambda_j}\right]} \tag{70}$$

Case 2

$$P_e = \lim_{N\to\infty} \sum_{j=1}^{N} \frac{1}{(1+\mu)\prod_{\substack{i=1\\ i\neq j}}^{N}\left(1+\mu\frac{\lambda_i^2}{\lambda_j^2}\right)\left(1-\frac{\lambda_i^2}{\lambda_j^2}\right)} \tag{71}$$

where $$\mu = 1 + \frac{E}{N_o}\,\frac{\pi}{2\beta}$$

A slightly different parameter that could be used in describing the performance of the system is the one-sided three db bandwidth "B" given as:

$$B = \frac{\alpha}{2\pi} = \frac{\beta}{\pi T} \text{ (cycles/second)} \tag{72}$$

In terms of this parameter, the two probabilities of error become:

Case 1:

$$Pe = \lim_{N\to\infty} \sum_{j=1}^{N} \frac{1}{2\left(1+\frac{5}{\pi}\frac{E}{N_o}\frac{\lambda_j}{BT}\right)\prod_{\substack{i=1\\ i\neq j}}^{N}\left[1+\left(1+\frac{10}{\pi}\frac{E}{N_o}\frac{\lambda_i}{BT}\right)\frac{\lambda_i}{\lambda_j}\right]\left[1-\frac{\lambda_i}{\lambda_j}\right]} \tag{73}$$

Case 2:

$$Pe = \lim_{N \to \infty} \sum_{j=1}^{N} \frac{1}{(1+\mu) \prod_{\substack{i=1 \\ i \neq j}}^{N} \left(1 - \mu \frac{\lambda_i^2}{\lambda_j^2}\right)\left(1 - \frac{\lambda_i^2}{\lambda_j^2}\right)} \tag{74}$$

where

$$\mu = 1 + \frac{E}{N_o (2BT)}$$

The performance curves to be presented give the probability of error versus E/N_o (measured in db.) for various values of BT.

7. COMPUTATION OF EIGENVALUES

The next step is the computation of the eigenvalues (λ_i's) to be substituted into Equations (70) and (71). These eigenvalues which satisfy the integral equation given by Equation (25) have been shown[12] to be related to the non-negative roots of the transcendental equations

$$\tan z = \beta/z \tag{75a}$$

$$\cot z = -\beta/z \tag{75b}$$

As can be seen from Figure 7, the smallest root, z_1, is a solution of (75a), the next smallest root, z_2, is a solution of (75b), etc., with the roots alternating between the two equations. The derived eigenvalues are related to the roots by the equation:

$$\lambda_i = \frac{\beta T}{\beta^2 + z_i^2} \tag{76}$$

Two checks that were used in the computation of these eigenvalues were:

(a) The sum of the eigenvalues should equal T

(b) The n^{th} root of Equation (75a) should fall in the interval $(n-1)\pi < z < (n-1/2)\pi$ while the n^{th} root of Equation (75b) should fall in the interval $(n-1/2)\pi < z < n\pi$. Furthermore, as n increases, the root occurs very close to the lower limit of allowed values.

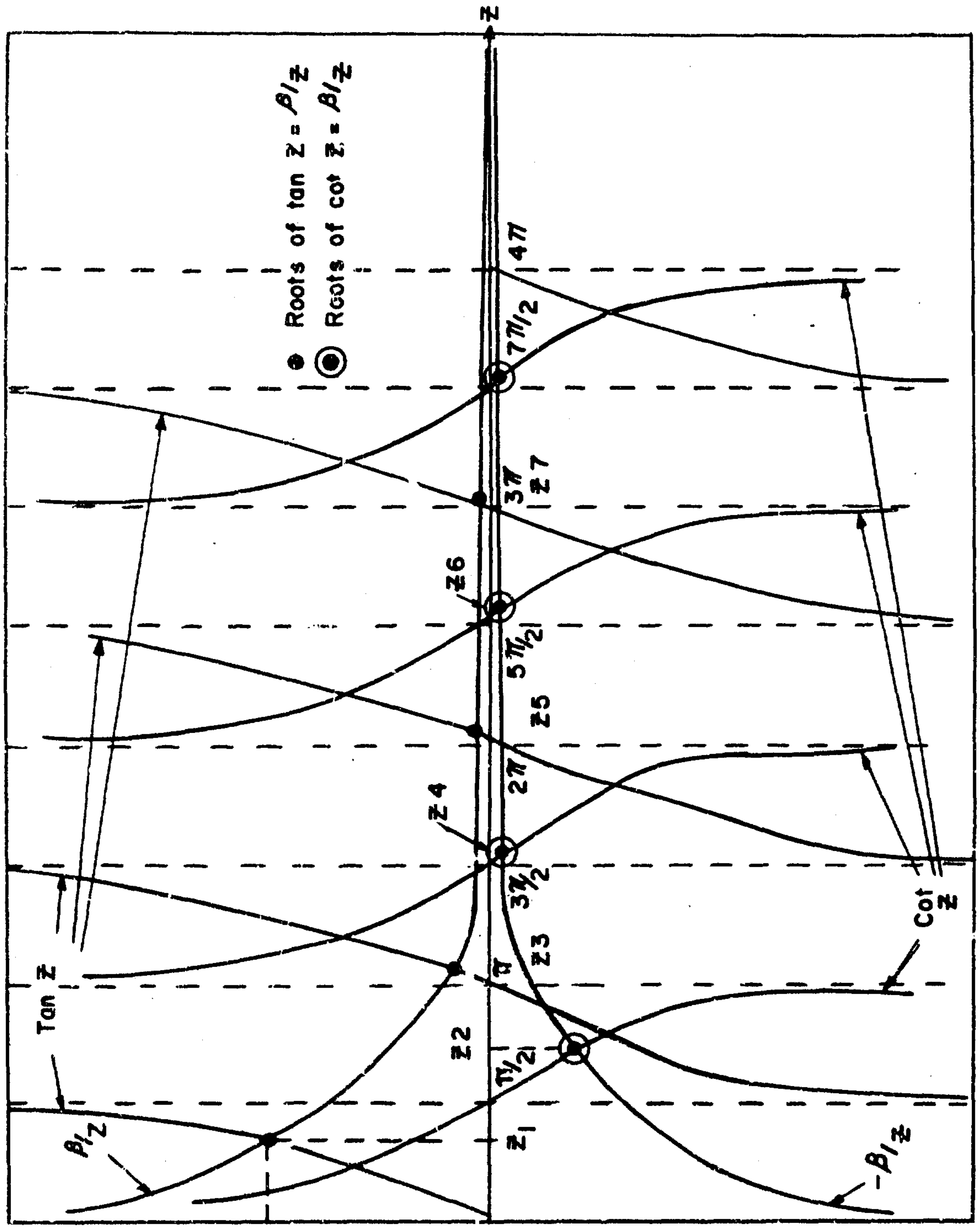


Figure 7

8. COMPUTATION OF ERROR PROBABILITIES

The mathematical forms of the computation [Equations (70), (71), (75), (76)] were programmed in FORTRAN IV. The programs for cases 1 and 2 are given in the Appendix. BT products of 0.5, 1.0, 2.0, 3.0, 4.0, and 5.0 were evaluated for E/N_o ratios of 6 db through 24 db in increments of two db. Tabulations of these outcomes follow with their corresponding graphs. (Figures 9 through 11.)

9. CALCULATIONS FOR OTHER FILTERS

The techniques outlined in this paper can be used for filter forms other than the RLC. In particular, if the noise processes $n_o(t)$ and $n_1(t)$, as given in equation (7), are such that

$$S_{x_i x_i}(\omega) = \mathcal{F}[R_{x_i x_i}(\tau)] = \begin{cases} \frac{S}{2B}, & |\omega| \leq 2\pi B \\ o, & |\omega| > 2\pi B \end{cases} \tag{77}$$

$$s_{y_i y_i}(\omega) = \mathcal{F}[R_{y_i y_i}(\tau)] = \begin{cases} \frac{S}{2B}, & |\omega| \leq 2\pi B \\ o, & |\omega| > 2\pi B \end{cases} \tag{78}$$

and if the receiver filters given in Figures 4 and 6 have transfer functions which have unity gain over a pass-band of bandwidth 2B (cycles/second) centered at ω_o and ω_1, and zero gain elsewhere, then the error probabilities given in Equations (73) and (74) still apply if the λ_i are the solutions to the integral equation

$$\int_o^T \frac{\sin 2\pi B\tau}{2\pi B\tau} \phi_i(\tau)\, d\tau = \lambda_i \phi_i(t) \quad o \leq t \leq T \tag{79}$$

Eigenvalues for this integral equation with a different normalization are given by Jacobs[8].

To arrive at an appropriate normalization consider the following. In the cases first examined a one-sided filter bandwidth was used exclusively, that is,

$$B = \frac{\alpha}{2\pi}$$

For the case of the flat spectrum, the double-sided bandwidth is used

$$B' = \frac{\alpha}{\pi} \tag{80}$$

PROBABILITY OF ERROR TABULATION

$\frac{E}{N_0}$ (db) \ BT=	0.5	1.0	2.0	3.0	4.0	5.0
6.00	1.059×10^{-1} *	1.149×10^{-1}	1.403×10^{-1}	1.642×10^{-1}	1.857×10^{-1}	2.079×10^{-1}
8.00	5.778×10^{-2}	5.751×10^{-2}	6.869×10^{-2}	8.319×10^{-2}	9.828×10^{-2}	1.136×10^{-1}
10.00	2.694×10^{-2}	6.840×10^{-3}	5.636×10^{-3}	6.206×10^{-3}	7.469×10^{-3}	9.251×10^{-3}
14.00	3.473×10^{-3}	1.528×10^{-3}	8.088×10^{-4}	7.119×10^{-4}	7.713×10^{-4}	9.242×10^{-4}
16.00	9.360×10^{-4}	2.468×10^{-4}	6.740×10^{-5}	3.973×10^{-5}	3.377×10^{-5}	3.534×10^{-5}
18.00	2.044×10^{-4}	2.815×10^{-5}	3.109×10^{-6}	9.988×10^{-7}	5.615×10^{-7}	4.475×10^{-7}
20.00	3.558×10^{-5}	2.206×10^{-6}	7.600×10^{-8}	1.065×10^{-8}	3.288×10^{-9}	1.714×10^{-9}
22.00	4.842×10^{-6}	1.155×10^{-7}	9.441×10^{-10}	4.604×10^{-11}	6.494×10^{-12}	1.916×10^{-12}
T =	$\pi/10$	$\frac{\pi}{5}$	$\frac{2\pi}{5}$	$\frac{3\pi}{5}$	$\frac{4}{5}\pi$	
BETA =	$\frac{\pi}{2}$	π	2π	3π	4π	5π
24.00	5.040×10^{-7}	3.964×10^{-9}	5.705×10^{-12}	7.765×10^{-14}	4.251×10^{-15}	6.522×10^{-16}

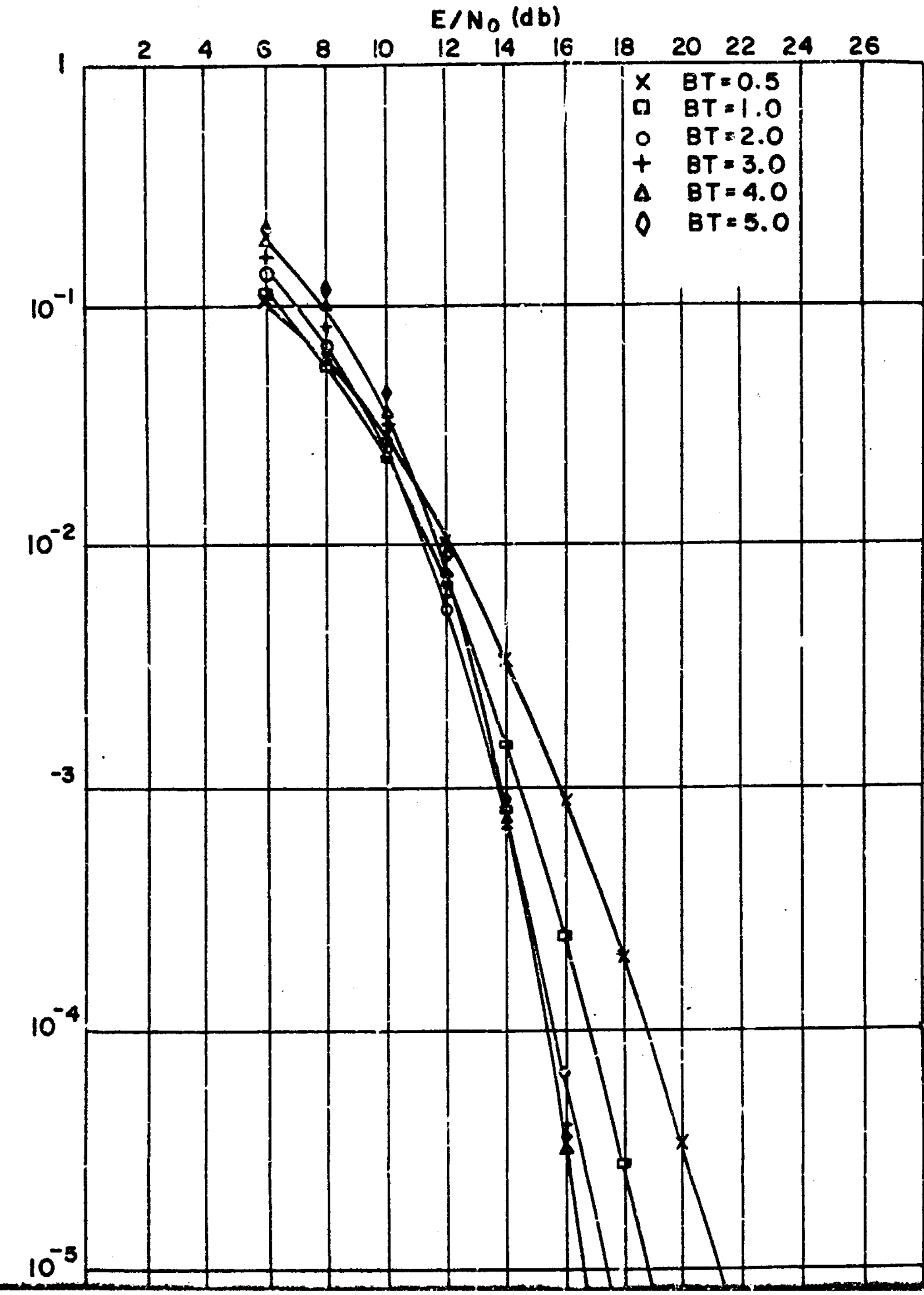
* Truncated Rather than Rounded

CASE 1. RLC EIGENVALUES

Figure 8

so that when we set $\alpha = 10$, then $B' = 10/\pi$. The flat spectrum eigenvalues, which are called λ_i's, are such that

$$\sum_i' \lambda'_i = BT \tag{81}$$



E/N0 (db)
2 4 6 8 10 12 14 16 18 20 22 24 26
1
10^-1
10^-2
-3
10^-4
10^-5
X BT=0.5
□ BT=1.0
○ BT=2.0
+ BT=3.0
Δ BT=4.0
◊ BT=5.0

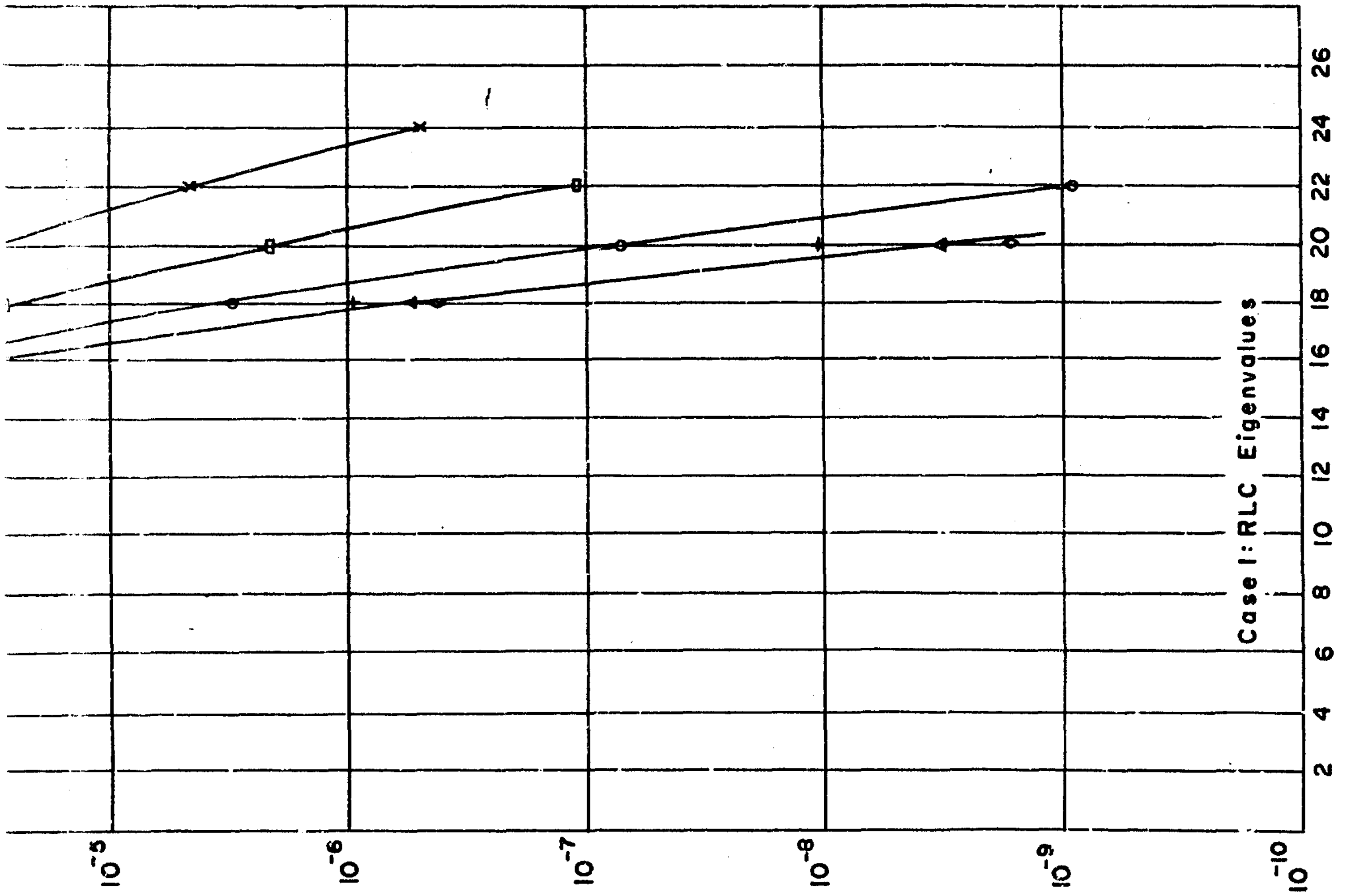

Case I: RLC Eigenvalues
2
4
6
8
10
12
14
16
18
20
22
24
26
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}

Figure 9

2

PROBABILITY OF ERROR TABULATION

E/N_o (db) \ BT =	0.5	1.0	2.0	3.0	4.0	5.0
6.	$7.527.10^{-2}$	$9.717\text{x}10^{-2}$	$1.310\text{x}10^{-1}$	$1.586\text{x}10^{-1}$	$1.818\text{x}10^{-1}$	$2.018\text{x}10^{-1}$
8.	$3.734\text{x}10^{-2}$	$4.573\text{x}10^{-2}$	$6.216\text{x}10^{-2}$	$7.809\text{x}10^{-2}$	$9.553\text{x}10^{-2}$	$1.115\text{x}10^{-1}$
10.	$1.572\text{x}10^{-2}$	$1.682\text{x}10^{-2}$	$2.099\text{x}10^{-2}$	$2.701\text{x}10^{-2}$	$3.417\text{x}10^{-2}$	$4.224\text{x}10^{-2}$
12.	$5.563\text{x}10^{-3}$	$4.695\text{x}10^{-3}$	$4.652\text{x}10^{-3}$	$5.572\text{x}10^{-3}$	$7.065\text{x}10^{-3}$	$9.089\text{x}10^{-3}$
14.	$1.636\text{x}10^{-3}$	$9.714\text{x}10^{-4}$	$6.312\text{x}10^{-4}$	$6.143\text{x}10^{-4}$	$7.125\text{x}10^{-4}$	$9.017\text{x}10^{-4}$
16.	[illegible]$.955\text{x}10^{-4}$	$1.460\text{x}10^{-4}$	$4.979.10^{-5}$	$3.281\text{x}10^{-5}$	$3.026\text{x}10^{-5}$	$3.400\text{x}10^{-5}$
18.	$7.758\text{x}10^{-5}$	$1.566\text{x}10^{-5}$	$2.193\text{x}10^{-6}$	$7.917\text{x}10^{-7}$	$4.863\text{x}10^{-7}$	$4.212\text{x}10^{-7}$
20.	$1.214\text{x}10^{-5}$	$1.170\text{x}10^{-6}$	$5.219\text{x}10^{-8}$	$8.213\text{x}10^{-9}$	$2.767\text{c}10^{-9}$	$1.574\text{x}10^{-9}$
22.	$1.488\text{x}10^{-6}$	$5.950\text{x}10^{-8}$	$6.485\text{x}10^{-10}$	$3.534\text{x}10^{-11}$	$5.390\text{x}10^{-12}$	$1.726\text{x}10^{-12}$
24.	$1.395\text{x}10^{-7}$	$1.092\text{x}10^{-9}$	$4.059\text{x}10^{-12}$	$6.136\text{x}10^{-13}$	$3.565\text{x}10^{-15}$	$5.657\text{x}10^{-16}$
T =	$\pi/10$	$\pi/5$	$\frac{2}{5}\pi$	$\frac{3}{5}\pi$	$\frac{4}{5}\pi$	π
BETA =	$\pi/2$	π	2π	3π	4π	5π

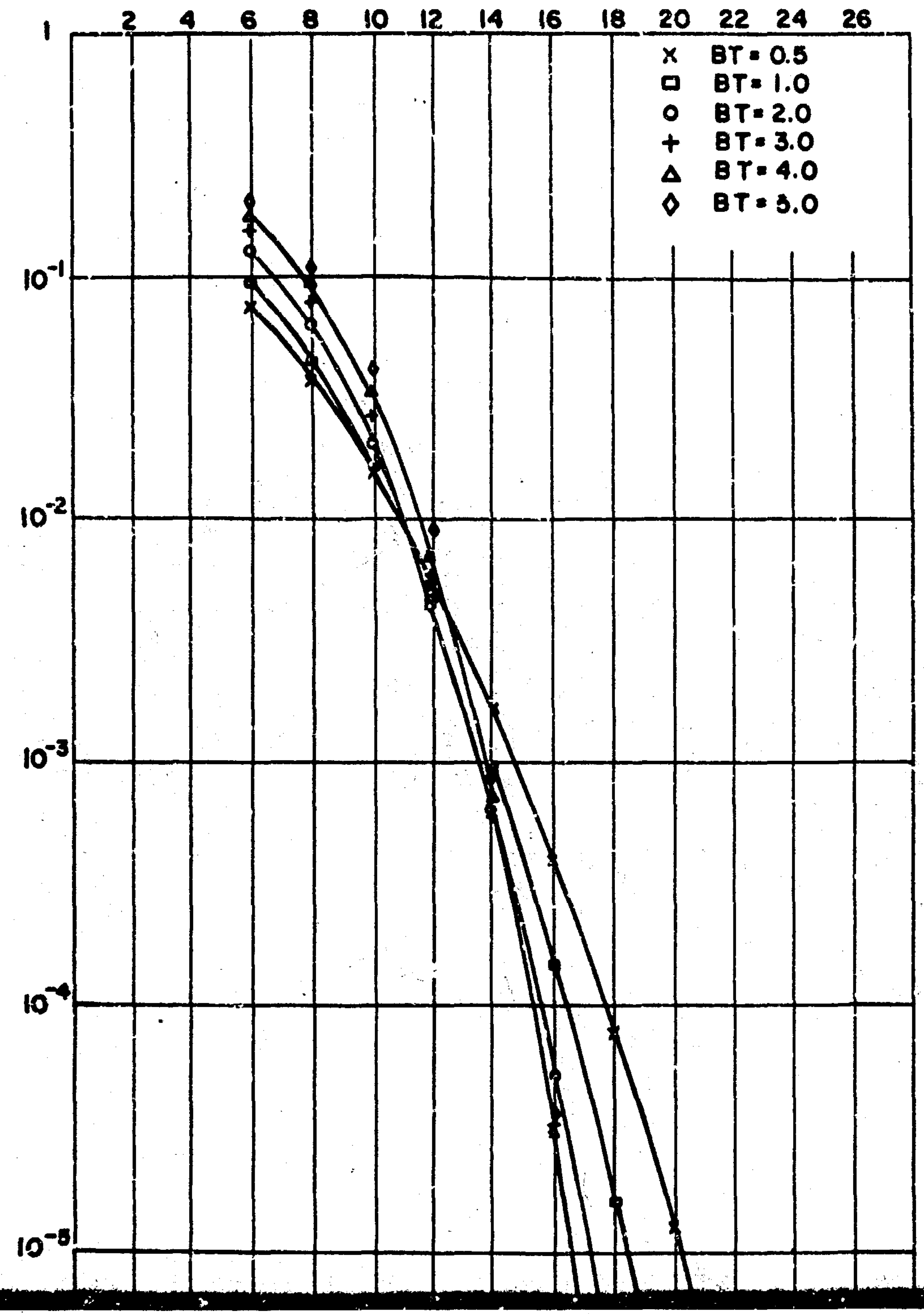
NOTE: Entries are truncated rather than rounded

CASE 2. RLC EIGENVALUES

Figure 10

We want $\sum_i \lambda_i = T$, so that we must let

$$\lambda_i = \frac{\lambda'_i}{B} \tag{82}$$



× BT = 0.5
□ BT = 1.0
○ BT = 2.0
+ BT = 3.0
△ BT = 4.0
◇ BT = 5.0
2
4
6
8
10
12
14
16
18
20
22
24
26
1
10⁻¹
10⁻²
10⁻³
10⁻⁴
10⁻⁵

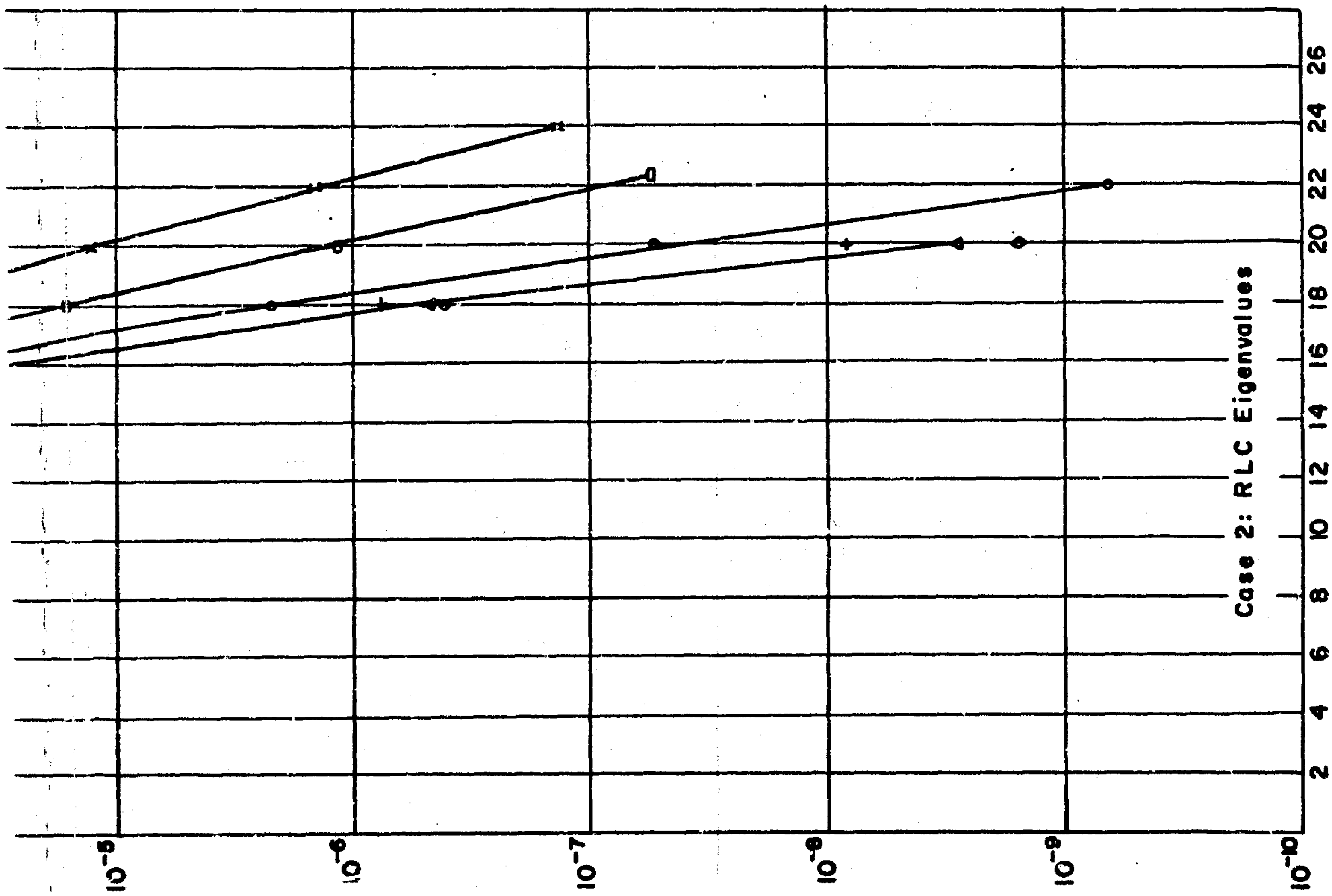

Case 2: RLC Eigenvalues
2
4
6
8
10
12
14
16
18
20
22
24
26
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}

Figure 11

2

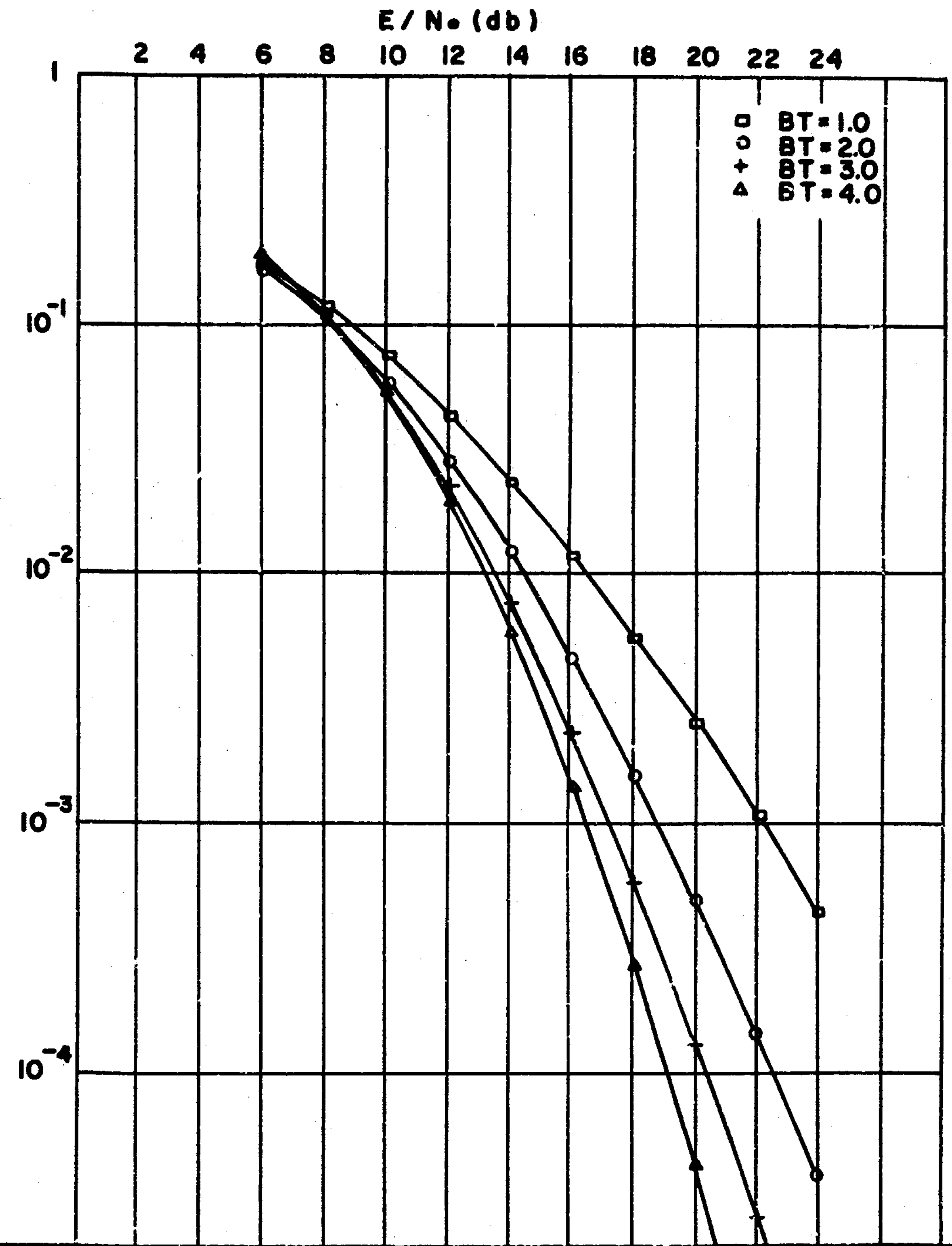
PROBABILITY OF ERROR TABULATION

E_b/N_0 (dB) \ BT=	1.0	2.0	3.0	4.0
6.	2.227×10^{-1}	2.536×10^{-1}	2.740×10^{-1}	2.871×10^{-1}
8.	1.617×10^{-1}	1.828×10^{-1}	1.990×10^{-1}	2.116×10^{-1}
10.	1.084×10^{-1}	1.172×10^{-1}	1.257×10^{-1}	1.340×10^{-1}
12.	6.696×10^{-2}	6.608×10^{-2}	6.707×10^{-2}	6.975×10^{-2}
14.	3.810×10^{-2}	3.260×10^{-2}	2.969×10^{-2}	2.877×10^{-2}
16.	2.005×10^{-2}	1.414×10^{-2}	1.088×10^{-2}	9.268×10^{-3}
18.	9.838×10^{-3}	5.444×10^{-3}	3.346×10^{-3}	2.348×10^{-3}
20.	4.544×10^{-3}	1.885×10^{-3}	8.787×10^{-4}	4.793×10^{-4}
22.	1.999×10^{-3}	5.961×10^{-4}	2.016×10^{-4}	8.124×10^{-5}
24.	8.471×10^{-4}	1.748×10^{-4}	4.135×10^{-5}	1.180×10^{-5}

NOTE: Entries are truncated rather than rounded.

CASE 2: FLAT SPECTRUM
EIGENVALUES

Figure 12


E/No (db)
2 4 6 8 10 12 14 16 18 20 22 24
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
BT = 1.0
BT = 2.0
BT = 3.0
BT = 4.0

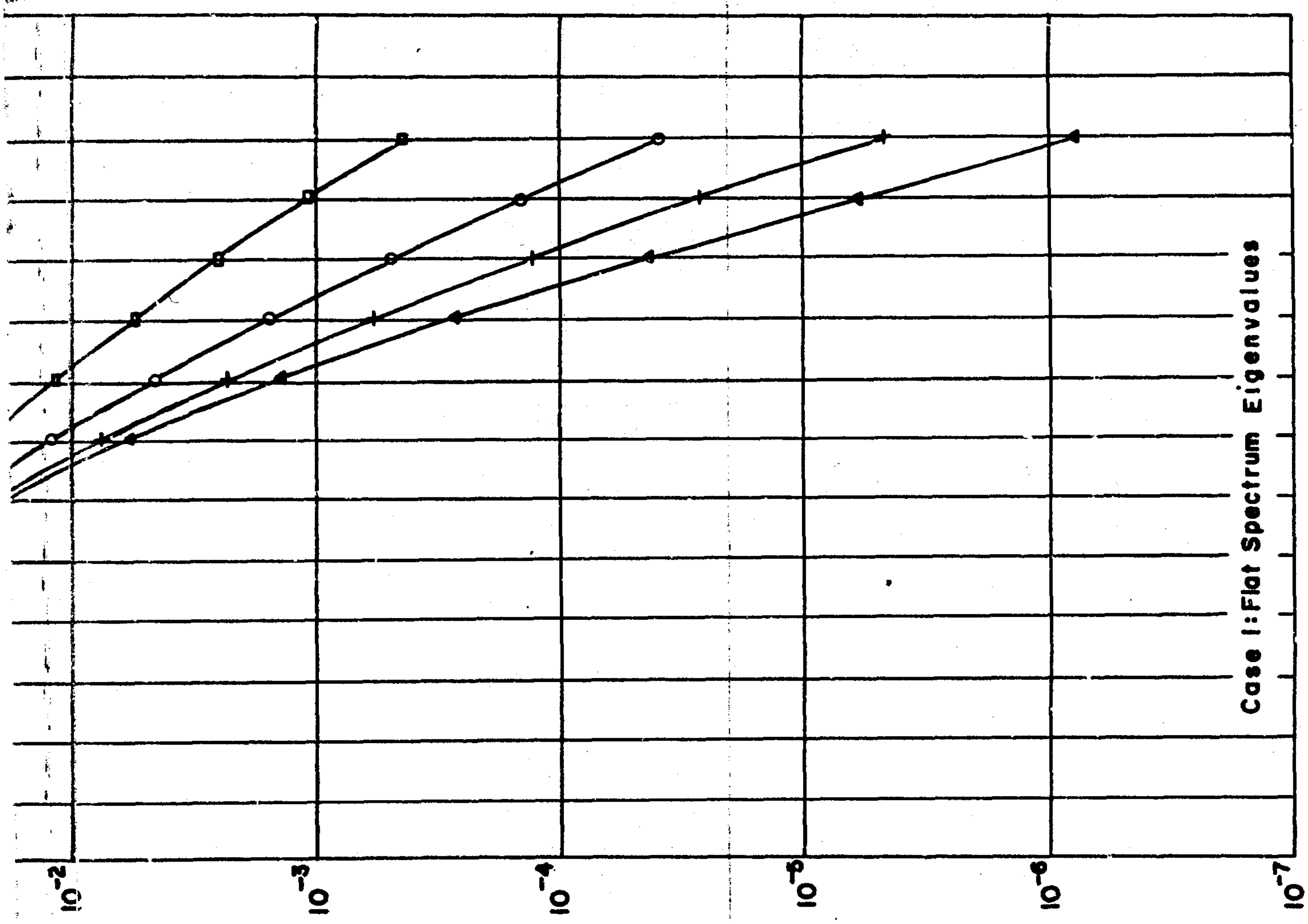


Figure 13

1

2

PROBABILITY OF ERROR TABULATION

$\frac{E}{N_0}$ (db) \ BT=	1.0	2.0	3.0	4.0
6.	1.717×10^{-1}	1.688×10^{-1}	1.752×10^{-1}	1.797×10^{-1}
8.	1.172×10^{-1}	1.057×10^{-1}	1.053×10^{-1}	1.068×10^{-1}
10.	7.397×10^{-2}	5.822×10^{-2}	5.321×10^{-2}	5.145×10^{-2}
12.	4.309×10^{-2}	2.814×10^{-2}	2.229×10^{-2}	1.951×10^{-2}
14.	2.323×10^{-2}	1.201×10^{-2}	7.779×10^{-3}	5.791×10^{-3}
16.	1.166×10^{-2}	4.569×10^{-3}	2.293×10^{-3}	1.366×10^{-3}
18.	5.491×10^{-3}	1.569×10^{-3}	5.829×10^{-4}	2.628×10^{-4}
20.	2.454×10^{-3}	4.931×10^{-4}	1.304×10^{-4}	4.255×10^{-5}
22.	1.052×10^{-3}	1.439×10^{-4}	2.624×10^{-5}	5.973×10^{-6}
24.	4.365×10^{-4}	3.960×10^{-5}	4.840×10^{-6}	7.467×10^{-7}

CASE 1: FLAT SPECTRUM

Figure 14

Thus, to get correct results, we must divide the flat spectrum eigenvalues by B', or

$$\lambda_i = (\lambda'_i) \frac{\pi}{10} \tag{83}$$

Note that when the flat spectrum eigenvalues are used, the normalization in Equation (83) will not make any difference since the eigenvalues always occur as $\lambda^2{}_i/\lambda^2{}_j$, thereby canceling the normalization. It will, however, make a difference in case one.

These eigenvalues normalized such that $\sum_i \lambda_i = T$ are listed in Table 7 (Appendix).

The results obtained using these eigenvalues for the two cases are given in Figures 13 and 15.

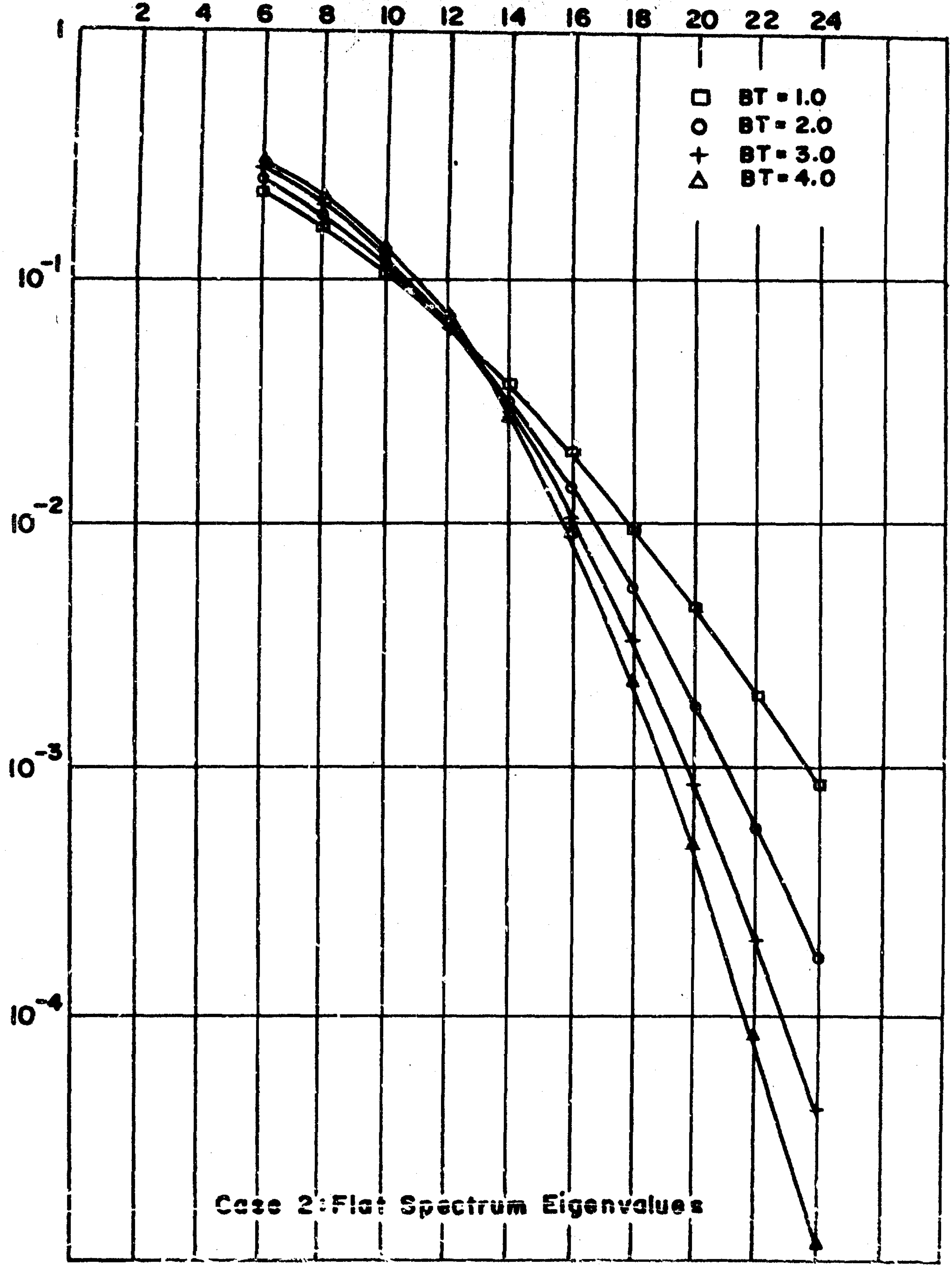
10. CONCLUSIONS

This report considers a sub-optimum receiver for binary frequency shift keyed (FSK) transmission which experiences fast fading and additive Gaussian white noise. The received signal is modeled as one of two narrow band Gaussian processes (whose center frequencies correspond to the frequencies used in the binary FSK transmission) corrupted by additive Gaussian white noise. The half-bandwidth of each narrow band process is B (cps) and the pulse duration is T (seconds).

A receiver is considered which passes the received waveforms through one of two narrow band filters (with center frequencies as above) and cross-correlates the outputs of these filters with the received waveform. The receiver makes a decision every T seconds as to which frequency was transmitted by choosing that frequency corresponding to the larger value of the cross-correlation. Two slightly different forms of this receiver are analyzed and curves for the probability of error versus the ratio of energy per bit to noise power density (E/N_0) for each receiver are presented with the product BT as a parameter. Two different spectra for the narrow band processes are studied.

The following conclusions were derived from the results. First, increasing BT does not give a constant improvement in probability of error for a fixed value of E/N_0. In fact, the rate of improvement diminishes as BT increases. It can further be noted that for E/N_0 greater than about 13 db a higher BT corresponds to a better system with the opposite effect for lower E/N_0. The results for BT = 0.5, 1., and 2. (for a particular spectrum) appear to be in essential agreement with those obtained experimentally by Cossette and Wolf[7] who used a slightly different receiver.

The results of this report would have practical importance if it were found that the receivers analyzed were easier to construct than the conventional receiver for this type of modulator and channel behavior. However, the practical aspects of the receiver design lay outside the realm of this study.



2
4
6
8
10
12
14
16
18
20
22
24
1
10⁻¹
10⁻²
10⁻³
10⁻⁴
BT = 1.0
BT = 2.0
BT = 3.0
BT = 4.0
Case 2: Flat Spectrum Eigenvalues

Figure 13

APPENDIX

```
C     ENERGY DETECTION EXTENSION
C     CROSS CORRELATION RECEIVER
C     (CASE 1)
C     RLC EIGENVALUES
C     PE MAGNITUDE, GT, SMALLEST EIGEN. N = 30
C     SINGLE PRECISION
C     BT = .5, 1., 2., 3., 4., 5.
C     PE VS EN0DB = 6, 8, 10, 12, 14, 16, 18, 20, 22, 24
C     FIND EIGENVALUES
      REAL LAMBDA
      DIMENSION LAMBDA (65), R(16), AT(20
      DATA (R(K), K = 1, 10)/.6, .8, 1., 1.2, 1.4, 1.6, 1.8,
      2., 2.2, 2.4/
      DATA (AT(KK), KK = 1, 6)/.5, 1., 2., 3., 4., 5./
      PI = 3.14159265
      DO 993 KK = 1, 6
      BT = AT (KK)
      BETA = PI * BT
      T = 0.62831853*BT
      PRINT 401, T, BETA, BT
401   FORMAT (2X, 3H T = E14.8, 2X, 6H BETA = E14.8,
      2X, 4H BT = E14.8)
      N = 30
      ACCURA = 1. E - 7
      DELTA = 0.5
      Z = 0.
      C1 = T * BETA
      C2 = BETA * BETA
      K2SIGN = 1
      K1SIGN = K2SIGN
      I = 0
      NNMAX = N/2
      PRINT 633
633   FORMAT (3X, 2H, I, 14X, 2H  Z, 14X, 10H LAMBDA(I))

      DO 30 NN = 1, NNMAX
3     DEL = DELTA
1     Z = Z + DEL
2     F1 = BETA/Z - SIN(Z)/COS(Z)
      IF(F1)4, 4, 6
4     L1SIGN = 0
      GO TO 8
6     L1SIGN = 1
8     IF (K1SIGN - L1SIGN)9, 1, 9
9     IF(ABS(DEL) - ACCURA)11, 11, 10
```

```
10      K1SIGN = L1SIGN
        DEL = -0. 1*DEL
        GO TO 2
11      I = I + 1
        LAMBDA(I) = C1/(C2 + Z*Z)
152     CONTINUE
645     PRINT 5050, I, Z, LAMBDA(I)
5050    FORMAT (2X, I5, 2E22.8)
644     CONTINUE
        K1SIGN = L1SIGN
23      DEL = DELTA
21      Z = Z+DEL
22      F2 = (COS(Z)/SIN(Z) ) + (BETA/Z)
        IF (F2) 24, 24, 26
24      L2SIGN = 0
        GO TO 28
26      L2SIGN = 1
28      IF(K2SIGN - L2SIGN)29, 21, 29
29      IF(ABS(DEL) - ACCURA)41, 41, 20
20      K2SIGN = L2SIGN
        DEL = -0. 1*DEL
        GO TO 22
41      I = I + 1
        LAMBDA(I) = C1/(C2 + Z * Z)
1645    PRINT 5050, I, Z, LAMBDA(I)
1644    CONTINUE
30      K2SIGN = L2SIGN
        SSUM = 0, 0
        DO 4049 L = 1, N
4049    SSUM = SSUM + LAMBDA(L)
        PRINT 9099, SSUM
9099    FORMAT (2X, 18H LAMBDA SUM EQUALS, E20. 8)
        D = N - 1
        TEST = D*PI/2
        PRINT 2020, TEST
2020    FORMAT (2X, 6H TEST = E20. 8)
C       CALCULATE PE
        PRINT 4036
4036    FORMAT (2X, 2H I, 14X, 5H GLOB)
        DO 501 K = 1, 10
        EN0DB = R(K)*10
        EN0 = 10. **R(K)
        SUM = 0.
        DO 900 I = 1, N
        GLOB = 1. E25
        ACON = 1. 0/LAMBDA(I)
```

```
            CHI = LAMBDA(I)/BETA
            SI = EN0*CHI
            W = 1. + 5. *SI
            WT = 1./W
            PROD = 1.
            DO 200 J = 1, N
            IF(J-I) 69, 200, 69
69          X = LAMBDA(J)*ACON
            CHJ = LAMBDA(J)/BETA
            SJ = EN0*CHJ
            V = 1. +10. *SJ
            VT = V*X
            VTH = 1. +VT
            XX = 1. -X
            PROD = PROD*VTH*XX
200         CONTINUE
            FKTR = 1./PROD
            GLOB = GLOB*WT*FKTR
999         PRINT 9090, I, GLOB
9090        FORMAT (2X, I5, E22. 8)
900         SUM = SUM + GLOB
899         PE = 0. 5*SUM*1. E-25
100         PRINT 111, PE, EN0DB, BETA
111         FORMAT (2X, 4H PE =, E20. 8, 5X, 7H EN0DB =,
            E20. 8, 5X, 6H BETA = E20. 8)
501         CONTINUE
993         CONTINUE
            STOP
            END
```

CORRELATION RECEIVER

```
C          ENERGY DETECTION EXTENSION
C          CROSS CORRELATION RECEIVER
C          RLC EIGENVALUES (CASE 2)
C          PE MAGNITUDE, GT, SMALLEST EIGEN. N = 30
C          SINGLE PRECISION
C          PE VS EN0DB = 6, 8, 10, 12, 14, 16, 18, 20, 22, 24
C          BT = .5, 1., 2., 3., 4., 5.
C          FIND EIGENVALUES
           REAL LAMBDA
           DIMENSION LAMBDA (40), BAMBDA (40), R(16), AT(10)
           DATA(R(K), K = 1, 10)/.6, .8, 1., 1.2, 1.4, 1.6, 1.8,
           2., 2.2, 2.4/
           DATA(AT(KK), KK = 1, 6)/.5, 1., 2., 3., 4., 5./
           PI  =  3.14159265
           DO 993 KK = 1, 6
           BT = AT(KK)
           BTT = 2. *BT
           BBTT = 1./BTT
           T = 0. 62831853*BT
           BETA = 5. *T
           PRINT 401, T, BETA, BT
401        FORMAT (2X, 3H T = E14. 8, 2X, 6H BETA = E14. 8,
           2X, 4H BT = E14. 8)
           N = 30
           ACCURA = 1. E-7
           DELTA = 0. 5
           Z = 0
           C1 = T*BETA
           C2 = BETA*BETA
           K2SIGN = 1
           K1SIGN = K2SIGN
           I = 0
           NNMAX = N/2
           PRINT 633
633        FORMAT (3X, 2H I, 14X, 2H Z, 14X, 10H LAMBDA(I),
           14X, 10H BAMBDA(I))
           DO 30 NN = 1, NNMAX
3          DEL = DELTA
1          Z = Z+DEL
2          F1 = BETA/Z-SIN(Z)/COS(Z)
           IF(F1)4, 4, 6
4          L1SIGN = 0
           GO TO 8
6          L1SIGN = 1
8          IF(K1SIGN-L1SIGN)9, 1, 9
9          IF(ABS(DEL) - ACCURA)11, 11, 10
```

```
10      K1SIGN = L1SIGN
        DEL = -0. 1*DEL
        GO TO 2
11      I = I + 1
        LAMBDA(I) = C1/(C2 + Z*Z)
        BAMBDA(I) = LAMBDA(I)*LAMBDA(I)
152     CONTINUE
645     PRINT 5050, I, Z, LAMBDA(I), BAMBDA(I)
5050    FORMAT (2X, I5, 3E22. 8)
644     CONTINUE
        K1SIGN = L1SIGN
23      DEL = DELTA
21      Z = Z+DEL
22      F2 = (COS(Z)/SIN(Z)) +(BETA/Z)
        IF(F2)24, 24, 26
24      L2SIGN = 0
        GO TO 28
26      L2SIGN = 1
28      IF(K2SIGN-L2SIGN)29, 21, 29
29      IF(ABS(DEL) - ACCURA)41, 41, 20
20      K2SIGN = L2SIGN
        DEL = -0. 1*DEL
        GO TO 22
41      I = I + 1
        LAMBDA(I) = C1/(C2 + Z*Z)
        BAMBDA(I) = LAMBDA(I)*LAMBDA(I)
1645    PRINT 5050, I, Z, LAMBDA(I), BAMBDA(I)
1644    CONTINUE
30      K2SIGN = L2SIGN
        SSUM = 0, 0
        DO 4049 L = 1, N
4049    SSUM = SSUM + LAMBDA(L)
        PRINT 9099, SSUM
9099    FORMAT (2X, 18H LAMBDA SUM EQUALS, E20. 8)
        D = N - 1
        TEST = D*PI/2.
        PRINT 2020, TEST
2020    FORMAT (2X, 6H TEST = E20. 8)
C       CALCULATE PE
        PRINT 4036
4036    FORMAT (2X, 2H I, 14X, 5H PROD)
        DO 501 K = 1, 10
        FNDB = R(K)*10.
        EN = 10. **R(K)
        SN = BBTT*EN
        SNDB = 10. *ALOG10(SN)
```

```
             U = 1. + SN
             UU = U-1.
             SUM = 0.
             DO 900 I = 1, N
      999    PRINT 9090, I, PROD
      9090   FORMAT (2X, I5, E22. 8)
             ACON = 1. 0/LAMBDA(I)
             PROD = 1. 0E35
             DO 200 J = 1, N
             IF(J-I) 69, 200, 69
      69     X = LAMBDA(J)*ACON
             Y = X*X
             PROD = PROD*1. 0/(1.0 + Y*(UU-U*Y))
      200    CONTINUE
      900    SUM = SUM + PROD
      899    PE = 1. 0/(1. 0 + U)*SUM*1. 0E-35
      100    PRINT 111, PE, ENDB, SNDB
      111    FORMAT (2X, 4H PE =, E20. 8, 5X, 6H ENDB =, E20.8
             5X, 6H SNDB =, E20.8)
      501    CONTINUE
      993    CONTINUE
             STOP
             END
```

CORRELATION RECEIVER

```
      C       FLAT SPECTRUM EIGENVALUES CASE 1
              REAL LAMBDA
              DIMENSION LAMBDA(50, R(16), AT(10)
              DATA(R(K), K = 1, 10)/.6, .8, 1., 1.2, 1.4, 1.6, 1.8,
              2., 2.2, 2, 4/
              PI = 3. 14159265
              DO 993 KK = 1, 4
              BT = KK
              BETA = PI*BT
              BTT = 2. *BT
              BBTT = 1./BTT
              READ 931, N
      931     FORMAT (I2)
              READ 246, (LAMBDA(I), I = 1. N)
      246     FORMAT (E[illegible], 8)
              PRINT 247, (LAMBDA(I), I = 1, N)
      247     FORMAT (E20. 8)
              SSUM = 0. 0
              DO 4049 L = 1, N
      4049    SSUM = SSUM + LAMBDA(L)
              PRINT 9099, SSUM
      9099    FORMAT (2X, 18H LAMBDA SUM EQUALS, E20. 8)
      C       CALCULATE PE
              PRINT 4036
      4036    FORMAT (2X, 2H I,  14X, 5H GLOB)
              DO 501 K = 1, 10
              EN0DB = R(K)*10.
              EN0 = 10   **R(K)
              SUM = 0
              DO 900 I = 1, N
              GLOB = 1. E25
              ACON = 1. 0/LAMBDA(I)
              CHI = LAMBDA(I)/BETA
              SI = EN0*CHI
              W = 1. +5. *SI
              WT = 1./W
              PROD = 1
              DO 200 J = 1, N
              IF(J-1) 69, 200, 69
      69      X = LAMBDA(J)*ACON
              CHJ = LAMBDA(J)/BETA
              SJ = EN0*CHJ
              V = 1. +10. *SJ
              VT = V*X
              VTH = 1. +VT
              XX = 1. -X
              PROD = PROD*VTH*XX
```

```
200     CONTINUE
        FKTR = 1,/PROD
        GLOB = GLOB*WT*FKTR
999     PRINT 9090, I, GLOB
9090    FORMAT (2X, I5, E22. 8)
900     SUM = SUM + GLOB
899     PE = 0. 5*SUM*1. E-25
100     PRINT 111, PE, EN0DB, BETA
111     FORMAT (2X, 4H PE =, E20. 8, 5X, 7H EN0DB =, E20 .8
        5X, 6H BETA =, E20. 8)
[illegible] CONTINUE
993     CONTINUE
        STOP
        END
```

CORRELATION RECEIVER

```
C          FLAT SPECTRUM EIGENVALUES CASE II
           REAL LAMBDA
           DIMENSION LAMBDA(50), R(16), AT(10)
           DATA(R(K), K = 1, 10)/.6, .8, 1., 1.2, 1.4, 1.6, 1.8,
           2., 2.2, 2.4/
           PI = 3.14159265
           DO 999 KK = 1, 4
           BT = KK
           BTT = 2.*BT
           BBTT = 1./BTT
           READ 931, N
931        FORMAT (I2)
           READ 246, (LAMBDA(I), I = 1, N)
246        FORMAT (E10. 8)
           PRINT 247, (LAMBDA(I), I = 1, N)
247        FORMAT (E20. 8)
           SSUM = 0.0
           DO 4049 L = 1, N
4049       SSUM = SSUM + LAMBDA(L)
           PRINT 9099, SSUM
9099       FORMAT (2X, 18H LAMBDA SUM EQUALS, E20. 8)
C          CALCULATE PE
           PRINT 4036
4036       FORMAT (2X, 2H  I, 14X, 5H PROD)
           DO 501 K = 1, 10
           ENDB = R(K)*10
           EN = 10.**R(K)
           SN = BBTT*EN
           SNDB = 10. *ALOG10(SN)
           U = 1. + SN
           UU = U - 1
           SUM = 0.
           DO 900 I = 1, N
999        PRINT 9090, I, PROD
9090       FORMAT (2X, I5, E22. 8)
           ACON = 1.0/LAMBDA(I)
           PROD = 1. 0E35
           DO 200 J = 1, N
           IF(J-I) 69, 200, 69
69         X = LAMBDA(J)*ACON
           Y = X*X
           PROD = PROD*1.0/(1.0 + Y*(UU-U*Y))
200        CONTINUE
900        SUM = SUM + PROD
899        PE = 1.0/(1.0 + U)*SUM*1.0E-35
100        PRINT 111, PE, ENDB, SNDB
```

```
111   FORMAT (2X, 4H PE =, E20. 8, 5X, 6H ENDB =, E20 .8
     5X, 6H SNDB =, E20. 8)
501   CONTINUE
993   CONTINUE
      STOP
      END
```

TABLE #1

RLC

T = 0.31415927E 00 BETA = 0.15707963E 01 BT = 0.50000000E 00

I	Z	LAMBDA(I)
1	0.10026742E 01	0.14210043E 00
2	0.21924764E 01	0.67838358E-01
3	0.35574032E 01	0.32632144E-01
4	0.50158793E 01	0.17862602E-01
5	0.65196132E 01	0.10972870E-01
6	0.80467656E 01	0.73415069E-02
7	0.95871783E 01	0.52285754E-02
8	0.11135709E 02	0.39019110E-02
9	0.12689531E 02	0.30183328E-02
10	0.14246978E 02	0.24020207E-02
11	0.15807011E 02	0.19557015E-02
12	0.17368951E 02	0.16225010E-02
13	0.18932335E 02	0.13673573E-02
14	0.20496839E 02	0.11677579E-02
15	0.22062227E 02	0.10087303E-02
16	0.23628326E 02	0.88001218E-03
17	0.25195006E 02	0.77438333E-03
18	0.26762165E 02	0.68664722E-03
19	0.28329724E 02	0.61298793E-03
20	0.29897621E 02	0.55055324E-03
21	0.31465806E 02	0.49717705E-03
22	0.33034237E 02	0.45119089E-03
23	0.34602883E 02	0.41129284E-03
24	0.36171714E 02	0.37645523E-03
25	0.37740708E 02	0.34585812E-03
26	0.39309846E 02	0.31884099E-03
27	0.40879111E 02	0.29486695E-03
28	0.42448488E 02	0.27349601E-03
29	0.44017967E 02	0.25436484E-03
30	0.45587536E 02	0.23717133E-03

LAMBDA SUM EQUALS 0.30738609E 00

TEST = 0.45553093E 02

TABLE #2

RLC

T = 0.62831853E 00   BETA = 0.31415927E 01   BT = 0.10000000E 01

I	Z	LAMBDA(I)
1	0.12046393E 01	0.17436293E 00
2	0.24744355E 01	0.12342841E 00
3	0.38287281E 01	0.80473722E-01
4	0.52515039E 01	0.52711125E-01
5	0.67204689E 01	0.35867098E-01
6	0.82190773E 01	0.25495331E-01
7	0.97368817E 01	0.18857350E-01
8	0.11267488E 02	0.14426516E-01
9	0.12806925E 02	0.11351750E-01
10	0.14352654E 02	0.91440984E-02
11	0.15902999E 02	0.75118293E-02
12	0.17456817E 02	0.62741821E-02
13	0.19013308E 02	0.53151622E-02
14	0.20571894E 02	0.45579462E-02
15	0.22132154E 02	0.39501974E-02
16	0.23693767E 02	0.34553553E-02
17	0.25256493E 02	0.30473026E-02
18	0.26820142E 02	0.27070064E-02
19	0.28384565E 02	0.24203462E-02
20	0.29949644E 02	0.21766766E-02
21	0.31515283E 02	0.19678546E-02
22	0.33081404E 02	0.17875688E-02
23	0.34647943E 02	0.16308684E-02
24	0.36214848E 02	0.14938274E-02
25	0.37782071E 02	0.13733013E-02
26	0.39349577E 02	0.12667478E-02
27	0.40917333E 02	0.11720940E-02
28	0.42485312E 02	0.10876368E-02
29	0.44053490E 02	0.10119663E-02
30	0.45621846E 02	0.94390765E-03

LAMBDA SUM EQUALS 0.60127003E 00

TEST = 0.45553093E 02

TABLE #3

RLC

T = 0.12566371E 01 BETA = 0.62831853E 01 BT = 0.20000000E 01

I	Z	LAMBDA(I)
1	0.13579462E 01	0.19107498E 00
2	0.27315071E 01	0.16820962E 00
3	0.41308029E 01	0.13964293E 00
4	0.55588752E 01	0.11218724E 00
5	0.70136998E 01	0.89045309E-01
6	0.84910380E 01	0.70764979E-01
7	0.99863744E 01	0.56719382E-01
8	0.11495776E 02	0.46003753E-01
9	0.13016102E 02	0.37796957E-01
10	0.14544938E 02	0.31452689E-01
11	0.16080456E 02	0.26490293E-01
12	0.17621274E 02	0.22559888E-01
13	0.19166340E 02	0.19407963E-01
14	0.20714850E 02	0.16850112E-01
15	0.22266182E 02	0.14751083E-01
16	0.23819850E 02	0.13010623E-01
17	0.25375468E 02	0.11553654E-01
18	0.26932730E 02	0.10323174E-01
19	0.28491389E 02	0.92755356E-02
20	0.30051243E 02	0.83768883E-02
21	0.31612128E 02	0.76007364E-02
22	0.33173907E 02	0.69261201E-02
23	0.34736465E 02	0.63363137E-02
24	0.36299709E 02	0.58178530E-02
25	0.37863556E 02	0.53598108E-02
26	0.39427938E 02	0.49532508E-02
27	0.40992796E 02	0.45908133E-02
28	0.42558080E 02	0.42663952E-02
29	0.44123745E 02	0.39749042E-02
30	0.45689754E 02	0.37120660E-02

LAMBDA SUM EQUALS 0.11490352E 01

TEST = 0.45553093E 02

TABLE #4

RLC

T = 0.18849556E 01 BETA = 0.94247780E 01 BT = 0.30000000E 01

I	Z	LAMBDA(I)
1	0.14211365E 01	0.19555373E 00
2	0.28481231E 01	0.18326397E 00
3	0.42856183E 01	0.16573185E 00
4	0.57364269E 01	0.14593646E 00
5	0.72015146E 01	0.12627414E 00
6	0.86804669E 01	0.10820817E 00
7	0.10172062E 02	0.92384499E-01
8	0.11674738E 02	0.78912660E-01
9	0.13186899E 02	0.67620420E-01
10	0.14707071E 02	0.58223103E-01
11	0.16233965E 02	0.50416849E-01
12	0.17766493E 02	0.43921874E-01
13	0.19303748E 02	0.38497909E-01
14	0.20844981E 02	0.33945997E-01
15	0.22389579E 02	0.30104549E-01
16	0.23937036E 02	0.26843562E-01
17	0.25486935E 02	0.24058830E-01
18	0.27038932E 02	0.21666828E-01
19	0.28592740E 02	0.19600454E-01
20	0.30148120E 02	0.17805607E-01
21	0.31704873E 02	0.16238460E-01
22	0.33262828E 02	0.14863337E-01
23	0.34821843E 02	0.13651039E-01
24	0.36381795E 02	0.12577538E-01
25	0.37942580E 02	0.11622954E-01
26	0.39504107E 02	0.10770752E-01
27	0.41066299E 02	0.10007106E-01
28	0.42629088E 02	0.93203967E-02
29	0.44192416E 02	0.87008141E-02
30	0.45756230E 02	0.81400325E-02

LAMBDA SUM EQUALS 0.16448637E 01

TEST = 0.45553093E 02

TABLE #5

RLC

T = 0.25132741E 01 BETA = 0.12566371E 02 BT = 0.40000000E 01

I	Z	LAMBDA(I)
1	0.14554862E 01	0.19735248E 00
2	0.29137500E 01	0.18979597E 00
3	0.43772054E 01	0.17835936E 00
4	0.58476472E 01	0.16440038E 00
5	0.73261580E 01	0.14926647E 00
6	0.88131605E 01	0.13406062E 00
7	0.10308560E 02	0.11955002E 00
8	0.11811911E 02	0.10618369E 00
9	0.13322568E 02	0.94163125E-01
10	0.14839801E 02	0.83522736E-01
11	0.16362873E 02	0.74197565E-01
12	0.17891082E 02	0.66072043E-01
13	0.19423789E 02	0.59011389E-01
14	0.20960425E 02	0.52879985E-01
15	0.22500489E 02	0.47551061E-01
16	0.24043548E 02	0.42910981E-01
17	0.25589228E 02	0.38860428E-01
18	0.27137205E 02	0.35313949E-01
19	0.28687204E 02	0.32198716E-01
20	0.30238985E 02	0.29452991E-01
21	0.31792343E 02	0.27024584E-01
22	0.33347102E 02	0.24869413E-01
23	0.34903106E 02	0.22950225E-01
24	0.36460225E 02	0.21235493E-01
25	0.38018341E 02	0.19698483E-01
26	0.39577354E 02	0.18310469E-01
27	0.41137176E 02	0.17070079E-01
28	0.42697729E 02	0.15942745E-01
29	0.44258945E 02	0.14920263E-01
30	0.45820763E 02	0.13990410E-01

LAMBDA SUM EQUALS 0.20911219E 01
TEST = 0.45553093E 02

TABLE #6

RLC

T = 0.31415927E 01 BETA = 0.15707963E 02 BT = 0.50000000E 01

I	Z	LAMBDA (I)
1	0.14770406E 01	0.19824712E 00
2	0.29556073E 01	0.19316129E 00
3	0.44370878E 01	0.18522095E 00
4	0.59226222E 01	0.17510626E 00
5	0.74130419E 01	0.16357016E 00
6	0.89088589E 01	0.15132422E 00
7	0.10410295E 02	0.13896375E 00
8	0.11917337E 02	0.12693599E 00
9	0.13429796E 02	0.11554220E 00
10	0.14947371E 02	0.10495916E 00
11	0.16469693E 02	0.95268123E-01
12	0.17996365E 02	0.86483033E-01
13	0.19526988E 02	0.78574225E-01
14	0.21061176E 02	0.71486416E-01
15	0.22598569E 02	0.65151455E-01
16	0.24138836E 02	0.59496772E-01
17	0.25681676E 02	0.54450669E-01
18	0.27226821E 02	0.49945374E-01
19	0.28774030E 02	0.45918596E-01
20	0.30323089E 02	0.42314147E-01
21	0.31873811E 02	0.39081971E-01
22	0.33426028E 02	0.36177855E-01
23	0.34979591E 02	0.33562960E-01
24	0.36534371E 02	0.31203293E-01
25	0.38090251E 02	0.29069161E-01
26	0.39647129E 02	0.27134649E-01
27	0.41204913E 02	0.25377143E-01
28	0.42763523E 02	0.23776920E-01
29	0.44322885E 02	0.22316715E-01
30	0.45882936E 02	0.20981445E-01

LAMBDA SUM EQUALS 0.24908018E 01

TEST = 0.45553093E 02

TABLE #7

λ_i(BT=1)	λ_i(BT=2)	λ_i(BT=3)
0.24610294E 00	0.30820594E 00	0.31381055E 00
0.64415216E-01	0.23550007E 00	0.30429152E 00
0.35732475E-02	0.76526055E-01	0.23017193E 00
0.67613357E-04	0.77430834E-02	0.82507648E-01
0.67770436E-06	0.33492519E-03	0.10966672E-01
0.42760217E-08	0.86126762E-05	0.69994684E-03
0.18594458E-10	0.15088441E-06	0.28416334E-04
	0.19268016E-08	0.82680435E-06
	0.18759706E-10	0.18234746E-07
		0.31610705E-09
		0.44243049E-11

λ_i(BT=4)
0.31414041E 00
0.31339271E 00
0.30140125E 00
0.22674445E 00
0.86290125E-01
0.13513561E-01
0.10927087E-02
0.58757207E-04
0.23453874E-05
0.73016896E-07
0.18285326E-08

FLAT SPECTRUM EIGENVALUES
NORMALIZED SO THAT $\sum_i \lambda_i = T$

(SEE EQUATION #83)

REFERENCES

1. Siegfried Reiger, "Error Probabilities of Binary Data Transmission Systems in the Presence of Random Noise," 1953 IRE Convention Record, Part 8, pp. 72-79.

2. John N. Pierce, "Diversity Improvement in Frequency-Shift Keying for Rayleigh Fading Conditions," AFCRC-TR-56-117, Air Force Cambridge Research Center, September 1956.

3. Jack K. Wolf, "On the Detection and Estimation Problem for Multi-dimensional Gaussian Random Channels," RADC-TR-61-214, Rome Air Development Center, November 1961.

4. Robert Price, "Error Probabilities for the Ideal Detection of Signals Perturbed by Scatter and Noise," Group Report 34-40, Lincoln Laboratories, October 3, 1955.

5. Stephen Rice, "Distribution of the Extreme Values of the Sum of N Sine Waves Phased at Random," Quarterly of Applied Mathematics, Vol. 12, pp. 375-381, January 1955.

6. Robert Price, "Some Thoughts on Modulation and Demodulation for the 'Needles' Channel, Part 1," Group Report 34-86, Lincoln Laboratories, April 15, 1959.

7. Edward Cossette and Jack Wolf, "Experimental Evaluation of an Optimum Detector for FSK Transmission Through a Rayleigh Fading Channel," RADC-TDR-62-554, Rome Air Development Center, November 1962.

8. Ira Jacobs, "Energy Detection of Gaussian Communication Signals," Tenth National Communications Symposium, October 1964, pg. 440-448.

9. Charles Meyer and Alfred Kobos, "Error Probability in Energy Detection of Gaussian Signals Through RC Filters," RADC-TR-66-144, Rome Air Development Center, July 1966.

10. Robert Price and Paul Green Jr., "Signal Processing in Radar Astronomy-Communication via Fluctuating Multipath Media," Technical Report No. 234, Lincoln Laboratories, October 6, 1960.

11. C. Helstrom, <u>Statistical Theory of Signal Detection,</u> Pergamon Press, C1960.

UNCLASSIFIED

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) RADC (EMCRS) Griffiss AFB NY 13440	2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED 2b. GROUP N/A
3. REPORT TITLE ON THE ANALYSIS OF A CROSS-CORRELATION RECEIVER FOR THE DETECTION OF NOISE-LIKE SIGNALS	
4. DESCRIPTIVE NOTES (Type of report and inclusive dates) In-House Report, July 1966 - July 1967 - Final	
5. AUTHOR(S) (First name, middle initial, last name) Jack K. Wolf Charles N. Meyer	

6. REPORT DATE October 1967	7a. TOTAL NO. OF PAGES 51	7b. NO. OF REFS 12
8a. CONTRACT OR GRANT NO. N/A b. PROJECT NO. 4519 Taks No. 451902 d.	9a. ORIGINATOR'S REPORT NUMBER(S) RADC-TR-67-485 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)	

10. DISTRIBUTION STATEMENT

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11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY Rome Air Development Center (EMCRS) Griffiss Air Force Base New York 13440

13. ABSTRACT

A common type of digital communication system is binary frequency shift keying (FSK) whereby every T seconds the transmitter sends a pulse of one of two frequencies. The receiver makes a decision (every T seconds) as to which frequency was transmitted. A sub-optimum receiver for this case obtains estimates of the two noise waveforms by passing received signals through filters centered at the sending frequencies and then crosscorrelates these estimates with the received waveform. Two slightly different versions of this cross-correlator were considered, and the probability of error for each case was calculated. The results seem to agree with previous experimental work by Cossette and Wolf.

DD FORM 1 NOV 65 1473

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14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Statistical Analysis Communications Signals Filters Correlation Techniques						

UNCLASSIFIED

Security Classification